IJA # 2550

Exam Materials from the Shamash Secondary School, 1954-1970


ELBA
RADO
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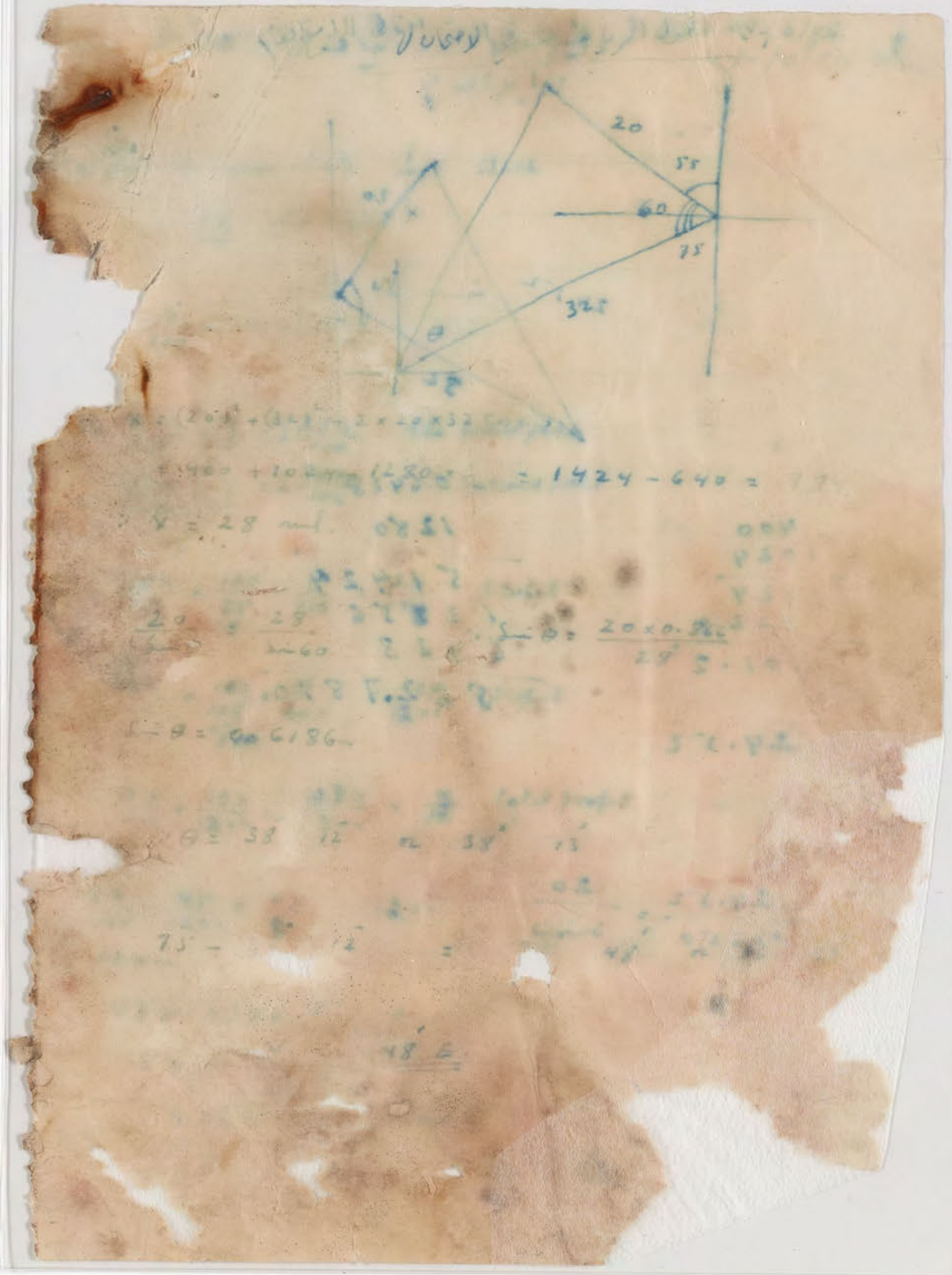
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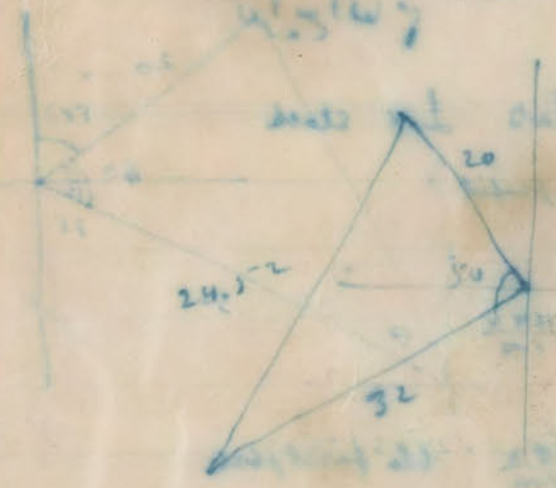


0.6428
1280
51424
12856
6428
822.7840

400
1024
1424

24.52

$\frac{24.52}{\sin 50} = \frac{20}{\sin}$

الرياضيات في ... للرابع العام

Let the man has £x stock

$\frac{4x}{100}$ his profit

$\frac{x}{100} \times 114 = £\frac{114x}{100}$

$\frac{114x}{100} \times \frac{1}{3} = £\frac{38x}{100}$ the first part

$\frac{114x}{100} \times \frac{2}{3} = £\frac{76x}{100}$ second part

$\frac{38x}{100} \times \frac{100}{95} \times \frac{5}{100} = \frac{2x}{100}$ profit

$\frac{76x}{100} \times \frac{100}{108} \times \frac{3}{400} = \frac{19x}{1200}$ profit

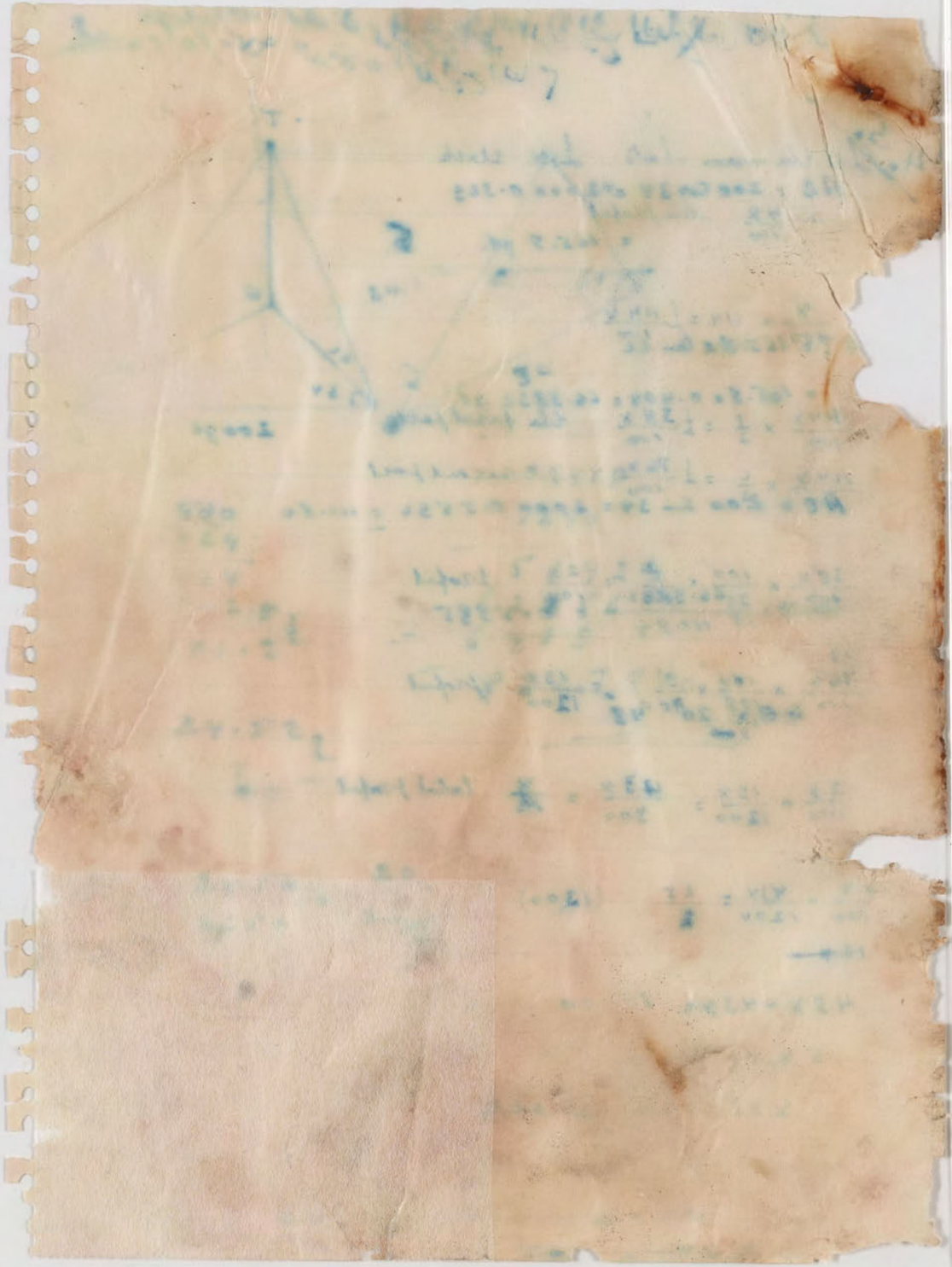
$\frac{2x}{100} + \frac{19x}{1200} = \frac{43x}{1200}$ total profit

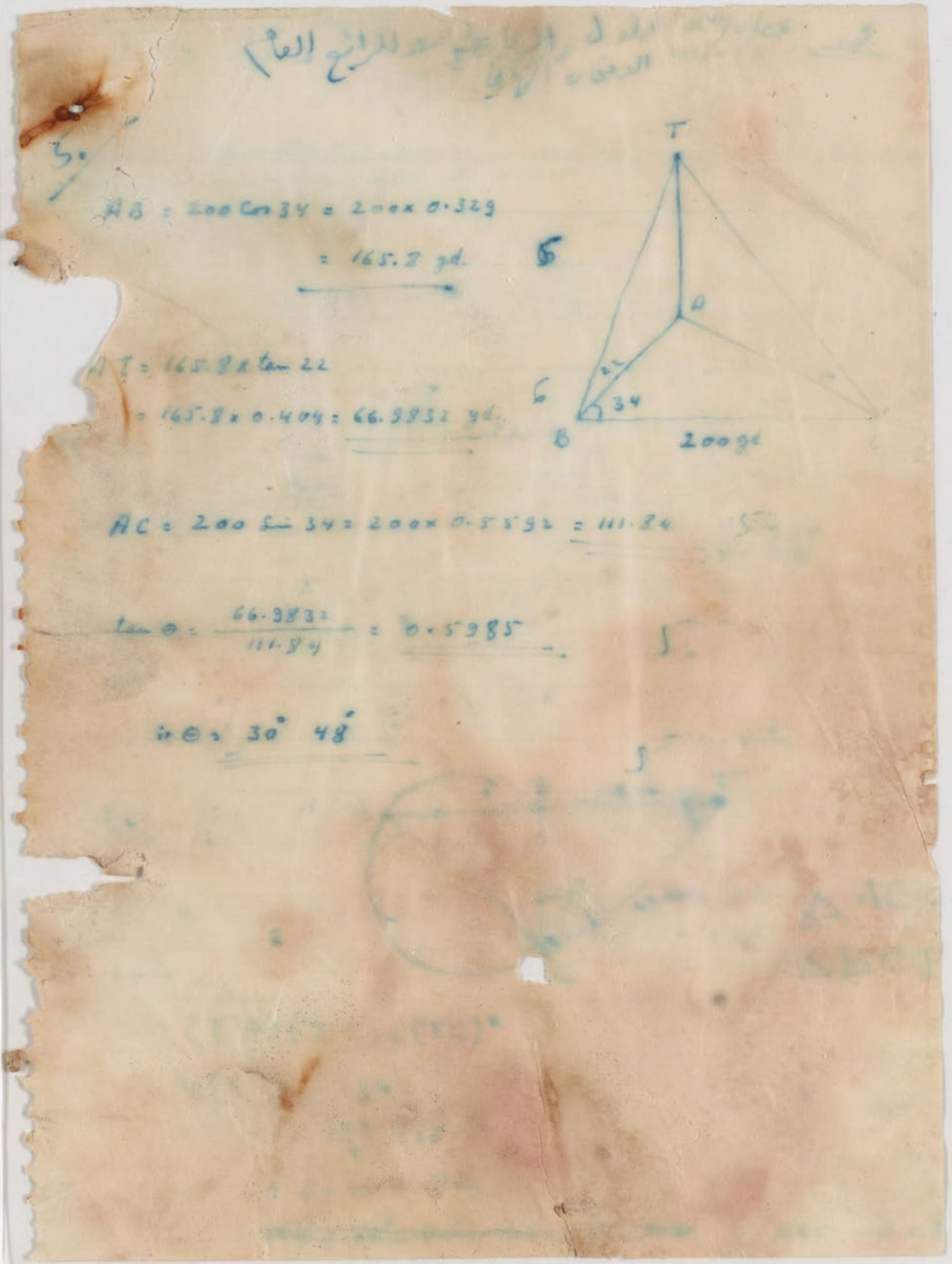
$\frac{4x}{100} - \frac{43x}{1200} = \frac{25}{2}$ (1200)

$48x - 43x = 15000$

$5x = 15000$

$x = 3000$ Stock





$AB = 200 \cos 34 = 200 \times 0.829$

$= 165.8$ yd.

$AT = 165.8 \times \tan 22$

$= 165.8 \times 0.404 = 66.9832$ yd.

$AC = 200 \sin 34 = 200 \times 0.5592 = 111.84$

$\tan \theta = \frac{66.9832}{111.84} = 0.5985$

$\therefore \theta = 30^\circ\ 48'$

SHAMASH SECONDARY SCHOOL

FINAL EXAMINATION - MAY 1970

Subject : Mathematics Date : 27/5/1970

Class : 4th year Secondary (scientific section) Time : 2 hours

Q1- A chord AB of a circle is produced to T. From T a line TC is drawn to touch the circle at C . If BT = 9 cm. and TC = 12 cm. Find the length of AT and the ratio of the areas of the triangles BTC and ATC and then prove $BC^2 : AC^2 = BT : AT$.

Q2- Two ships leave the same port at the same time and steam at 10 and 16 m.p.h. respectively in the direction N. 55 W. and S. 75 W. Find their distance apart after 2 hours and the bearing of the first ship to the second .

Q3- PQ , CD are parallel chords of a circle , the tangent at D cuts PQ produced at T , B is a point of contact of the other tangent from T ; prove that BC bisect PQ .

Q4- A man has a certain amount of 4 % stock . He sells it at 114 and invests one - third the proceed in 5% at 95 , and the rest in 2¼ % stock at 108. He then finds that his annual income is reduced by £ 12 10s. . Find the amount of the original stock had he .

Q5- ABC is a triangle in a horizotal plane $ BC = 120 yd. , angle BAC = 90 and angle ABC = 34 . At A is a vertical pole AT , the angle of elevation of T from B = 22 , calculate the height of AT , and the angle of elevation of T from C ,

SHAMASH SECONDARY SCHOOL
itional Examination, September, 1969.

Subject: Algebra
Class : 4th Year, Secondary

Date : 5/9/1969.
Time : 8:00-11:00 a.m.

Attempt all questions:

1. (i) Find the values of 'a' and 'b' if $3x^3 - ax + b$ is exactly divisible by $(x+1)(x-2)$. If 'a' and 'b' have these values, factor the expression completely. (10 marks).

(ii) If $x^3 + 3x + 5 = x(x+1)(x+2) + Ax(x+1) + Bx + C$ for all values of x, find the values of the constants A, B, and C. (10 marks).

2. (i) The 15th term of an arithmetic progression is 25 and the sum of the first 10 terms is 60. Find the first term of the progression, the common difference and the sum of the first 16 terms. (10 marks).

(ii) $p+3$, $p+8$, and $p+18$, are the 3rd, 4th, and 5th terms of a geometric progression. Find the value of p. Find also the common ratio and the 9th term of the progression. (10 marks).

3. A can walk a mile in 2 minutes less time than B would take. In a walking race, B has a start of ¼ mile, and A overtakes B in 10 minutes. Assuming that both men walk at a uniform rate, find their rates of walking in miles per hour. (20 marks).

4. Find the value of x from the following equation :

$10^{2x} - 11(10^x) + 10 = 0$.. (10 marks).

(ii) Use logarithms to compute the value of the following expression :

$$\sqrt[7]{\frac{(0.002013)^2(\sin 15^\circ\ 12')^3}{(4.004)^3(\cos 42^\circ\ 13')^2}}$$ (10 marks).

5. (i) Plot the curve of the equation $y = 2x^2 - x - 3$ at half-unit intervals between $x = -1.5$ and $x = 2$, choosing one inch as one unit on each of the two axes. (8 marks).

(ii) From your graph, find the roots of the equation $x + 3 = 2x^2$. marks).

(iii) Find the values of x for which the function $2x^2 - x -$ is always positive. 4 marks).

(iv) By drawing another straight line on your diagram, find the roots of the equation $2x^2 - x$ $1 = 0$. 4 marks).

SHAMASH SECONDARY SCHOOL
Conditional Examination, September, 1969.

Subject: Algebra — Date : 5/9/1969.
Class : 4th Year, Secondary — Time : 8:00-11:00 a

Attempt all questions:

1. (i) Find the values of 'a' and 'b' if $3x^3 - ax + b$ is exactly divisible by $(x+1)(x-2)$. If 'a' and 'b' have these values, factor the expression completely. (10 marks).

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4. Find the value of x from the following equation :

   $10^{2x} - 11(10^x) + 10 = 0.$ (10 marks).

   (ii) Use logarithms to compute the value of the following expression :

$$\sqrt[7]{\frac{(0.002013)^2(\sin 15^\circ\ 12\frac{1}{2}')^3}{(4.004)^3(\cos 42^\circ\ 13\frac{1}{3}')^2}}$$

(10 marks).

5. (i) Plot the curve of the equation $y = 2x^2 - x - 3$ at half-unit intervals between $x = -1.5$ and $x = 2$, choosing one inch as one unit on each of the two axes. (8 marks).

   (ii) From your graph, find the roots of the equation $x + 3 = 2x^2$. (4 marks).

   (iii) Find the values of x for which the function $2x^2 - x - 3$ is always positive. (4 marks).

   (iv) By drawing another straight line on your diagram, find the roots of the equation $2x^2 - x - 1 = 0$. (4 marks).

Shamash Secondary School
Final Examination May 1969

Subject: Algebra
Class: 4th Year, Scientific

Date: 14/5/1969
Time: 8:00 - 11:00 a.m.

Answer all questions:

1. (i) In the expression $x^3+Ax^2+31x+B$, A and B are constant. Find the values of A and B which will make this expression divisible by (x-2) (x-3) and find the remaining factor.
(10 marks)

(ii) The wages of 12 men and 7 boys amount to £9 13s. If 3 men together receive 8s. more than 4 boys, what are the wages of each man and boy ?
(10 marks)

2. (i) Solve the equation $(\log x)^2 = \log x^7 - 10$, finding two values for x.
(10 marks)

(ii) Solve the two simultaneous equations:

$$3x^2+xy-2y^2+7=0 \quad \ldots\ldots\ldots (1)$$
$$x^2-xy+y^2-7=0 \quad \ldots\ldots\ldots (2)$$
(10 marks)

3. (i) Find the value of x from the following equation without using the tables:

$$(5)(4^{3x-1})(\sqrt{8})^{1-x} = (\sqrt{2})^{x}(\sqrt{50})$$
(10 marks)

(ii) Compute the value of $\sqrt[7]{\dfrac{(0.5002)^2 \sin^3 14^\circ 25'}{(4.003)^3 \cos^2 15^\circ 27'}}$ (10 marks)

4. (i) If $(b+c)^{-1}$, $(c+a)^{-1}$, $(a+b)^{-1}$ are in arithmetical progression, prove that a^2, b^2, c^2 are also in arithmetical progression.
(10 marks)

(ii) A bouncing tennis ball rebounds each time to a height one half the height of the previous bounce. If it is dropped from a height of 10 ft., show:

(a) that the total distance it has travelled when it hits the ground for the 10th time is equal to $29\frac{123}{128}$ ft. (b) Show also that the total distance it travels before coming to rest is 30 ft.
(10 marks)

(cont'd.p.2)..

(cont'd)..

Algebra 4th Year. Scientific 14/5/1969.

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5. (i) Plot the curve of the function $3+2x-x^2$ for values of x from x=-2 to x=4, choosing one half of an inch for each unit on the axis of x and on the axis of y.

(4 marks)

(ii) From your graph, find the roots of the equation $x^2-3=2x$.

(3 marks)

(iii) Find the values of x for which the function $3+2x-x^2$ is always positive.

(3 marks)

(iv) Find from your diagram the value of x at which the function $3+2x-x^2$ is greatest and state the maximum value.

(3 marks)

(v) By plotting another curve on the same diagram, find the values of x for which $3+2x-x^2 > \frac{x}{2} + 2$. (4 marks)

(vi) From your last diagram, find the roots of the equation $3+2x-x^2 = \frac{x}{2} + 2$.

(3 marks)

Shamiash Secondary School
Final Examinations May 1969

Solution to Algebra Question, 4th Year 14/5/1969.

1. (i) Divide by $(x-2)$: $x-2 \mid x^3+Ax^2+31x+B \mid x^2+(A+2)x+2A+35$

x^3-2x^2

$(A+2)x^2+31x+B$

$(A+2)x^2-2(A+2)x$

$(2A+4+31)x+B$

or $(2A+35)x+B$

$(2A+35)x-4A-70$

$4A+B+70=0 \quad \therefore B=-4A-70$ ..... (1)

Now Divide the Quotient by $(x-3)$: $x-3 \mid x^2+(A+2)x+2A+35 \mid x+A+5$

x^2-3x

$(A+5)x+2A+35$

$(A+5)x-3A-15$

$5A+50$: $5A+50=0$ ..... (2)

$\therefore 5A+50=0 \quad \therefore A=-\frac{50}{5}=-10$

$\therefore B=-4(-10)-70=-30$

$\therefore$ the 3rd Factor is $= x+A+5 =$

$= x-10+5 = x-5$

$\therefore A=-10$, $B=-30$, 3rd factor $= x-5$ } Ans.

and the expression is: $x^3-10x^2+31x-30=(x-2)(x-3)(x-5)$

An alternative method: Using the remainder theorem, when $x=2$ then

$2^3+2^2A+2\times31+B=0 \quad \therefore 4A+B=-70$ ..... (1) also when $x=3$, then

also $3^3+3^2A+3\times31+B=0 \quad \therefore 9A+B=-120$ ..... (2)

$5A=-50 \quad \therefore A=-10 \quad \therefore$ from (1): $4(-10)+B=-70 \quad \therefore B=-30$

$\therefore A=-10$, $B=-30$ } Ans. Factoring, we get $x^3-10x^2+31x-30=(x-2)(x-3)(x-5)$ Ans. 2

(ii) 12 men + 7 boys = £9 13s. also 3 men = 4 boys + 8s.

Let x shillings be the wages of one boy

y " " " " " " man

$\therefore 12y+7x=9\times20+13$ or $12y+7x=193$ ..... (1)

also $3y=4x+8$ or $3y-4x=8$ ..... (2)

$\therefore 12y+7x=193$ ..... (1)

$12y-16x=32$ ..... (2)

$\therefore 23x=161 \quad \therefore x=\frac{161}{23}=7$ shillings

$\therefore y=\frac{4x+8}{3}=\frac{28+8}{3}=\frac{36}{3}=12$s.

$\therefore$ wages of one boy $= x$ shill. $= 7$s.

" " " man $= y$ shill $= 12$s. } Ans.

2 (i) Solve the equation: $(\log x)^2 - 7\log x + 10 = 0$

$\therefore (\log x - 2)(\log x - 5) = 0$

$\therefore \log x - 2 = 0$ or $\log x = 2$ $\therefore x = 10^2 = 100$ Ans. 1

or $\log x - 5 = 0$ or $\log x = 5$ Ans. 2

(ii) $3x^2 + xy - 2y^2 + 7 = 0$ ... ①

$x^2 - xy + y^2 - 7 = 0$ ... ② } adding, we get:

$4x^2 - y^2 = 0$ $\therefore (2x+y)(2x-y) = 0$

$\therefore$ either $y = 2x$ ... ③ | from ② & ③ : $x^2 - x(2x) + (2x)^2 - 7 = 0$

or $y = -2x$ ... ④ | or $x^2 - 2x^2 + 4x^2 = 7$ or $3x^2 = 7$ $\therefore x = \pm\sqrt{\frac{7}{3}}$

$\therefore y = 2x$ $\therefore$ $y = \pm 2\sqrt{\frac{7}{3}}$

From ② & ④ : $x^2 - x(-2x) + (-2x)^2 - 7 = 0$ or $x^2 + 2x^2 + 4x^2 = 7$

or $7x^2 = 7$ $\therefore x^2 = 1$ $\therefore x = \pm 1$

$\therefore y = -2x$ $\therefore y = -2(\pm 1) \therefore y = \mp 2$

$\therefore$ $x = \sqrt{\frac{7}{3}}$, $y = 2\sqrt{\frac{7}{3}}$ } Ans. 1 $\quad$ $x = -\sqrt{\frac{7}{3}}$, $y = -2\sqrt{\frac{7}{3}}$ } Ans. 2

$x = 1$, $y = -2$ } Ans. 3 $\quad$ $x = -1$, $y = 2$ } Ans. 4

3 (i) $(5)(4^{3x-1})(\sqrt{8})^{1-x} = (\sqrt{2})^x(\sqrt{5^x})$

$5 \times 2^{2(3x-1)} \times 2^{\frac{3}{2}(1-x)} = 2^{\frac{x}{2}} \times 5^x \times 2^{\frac{1}{2}}$

$\therefore 2^{6x-2} \times 2^{\frac{3}{2}-\frac{3x}{2}} = 2^{\frac{x+1}{2}}$ $\therefore 2^{6x-2+\frac{3}{2}-\frac{3x}{2}} = 2^{\frac{x+1}{2}}$

$\therefore 6x - 2 + \frac{3}{2} - \frac{3x}{2} = \frac{x+1}{2}$ $\therefore \frac{9x}{2} - \frac{1}{2} = \frac{x}{2} + \frac{1}{2}$

$\therefore 4x = 1$ $\therefore x = \frac{1}{4}$ Ans.

3(ii) $x = \sqrt[7]{\dfrac{(0.5002)^2 \sin^3 14^\circ 25'}{(4.003)^3 \cos^2 15^\circ 27'}}$

$\log 0.5002 = \bar{1}.6992$
$\log \sin 14^\circ 25' = \bar{1}.3962$
$\log 4.003 = 0.6024$
$\log \cos 15^\circ 27' = \bar{1}.9840$

$2\log 0.5002 = \bar{1}.3984$
$3\log \sin 14^\circ 25' = \bar{2}.1886$
$\log \text{Num} = \bar{3}.5870$
$\log \text{Den} = 1.7752$
$7\log x = \bar{5}.8118$
$\log x = \bar{1}.4017$
$x = 0.2522$ or 2.522×10^{-1}

$3\log 4.003 = 1.8072$
$2\log \cos 15^\circ 27' = \bar{1}.9680$
$\log \text{Den.} = 1.7752$

Ans.

(i) If $(b+c)^{-1}$, $(c+a)^{-1}$, $(a+b)^{-1}$ are in A.P., prove that a^2, b^2, c^2 are also in A.P.

By hypothesis: $(a+b)^{-1} - (c+a)^{-1} = (c+a)^{-1} - (b+c)^{-1}$ or

$$\frac{1}{a+b} - \frac{1}{c+a} = \frac{1}{c+a} - \frac{1}{b+c}$$ or

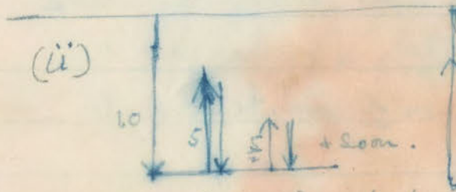
$$\frac{c+b-a-b}{(a+b)(c+a)} = \frac{b+c-c-a}{(c+a)(b+c)}$$ or $$\frac{c-b}{(a+b)(c+a)} = \frac{b-a}{(c+a)(b+c)}$$

$\therefore \dfrac{c-b}{a+b} = \dfrac{b-a}{b+c}$ or $(c-b)(c+b) = (b-a)(b+a)$ or

$\therefore c^2 - b^2 = b^2 - a^2$ which makes a^2, b^2, c^2 in A.P. by definition of A.P.

Q.E.D.

(ii)



② Total distance it travels before coming to rest =
$= S_{n\to\infty} = 10 + (10 + 5 + \frac{5}{2} + \dots \text{ to infinity})$
$= 10 + \left(\frac{10}{1-\frac{1}{2}}\right) = 10 + \frac{10}{\frac{1}{2}} = 10 + 20 = 30$ ft.
Ans. 2

① when the ball has struck the ground for the first time it has travelled 10 ft. When it strikes the ground for the 2nd time it will have gone upward 5 ft & downward 5 ft also
∴ it will have travelled another 10 ft, the next distance will be 5 ft + so on.
∴ the total distance travelled by the end of the 10th time it strikes the ground will be $10 + 10 + 5 + \frac{5}{2} + \frac{5}{4} + \dots$ to ten terms $= 10 + (10 + 5 + \frac{5}{2} + \frac{5}{4} + \dots$ to 9 terms

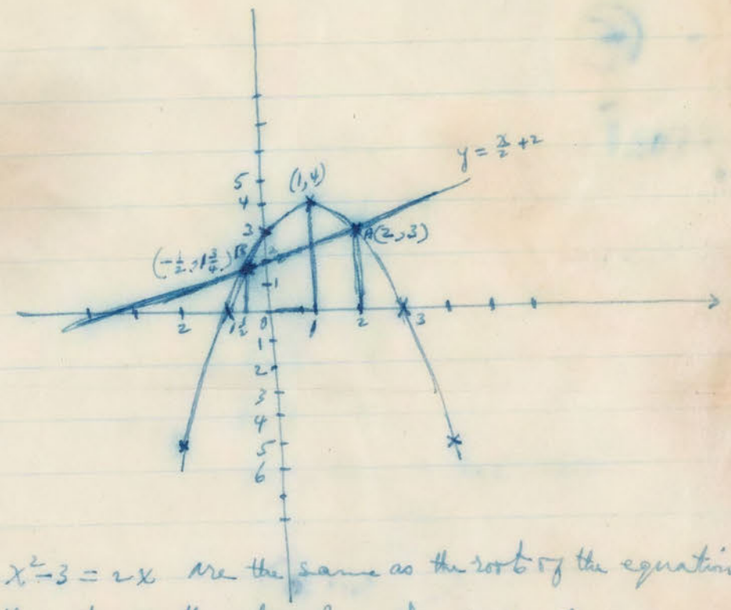
∴ total distance $= 10 + \dfrac{10\left(1 - \left(\frac{1}{2}\right)^9\right)}{1-\frac{1}{2}} = 10 + \dfrac{10\left(1 - \frac{1}{512}\right)}{\frac{1}{2}} = 10 + 20\left[1 - \frac{1}{512}\right] = 10 + 20\left(\frac{511}{512}\right)$

$= 10 + \frac{10220}{512} = 10 + 19\frac{123}{128} = 29\frac{123}{128}$ ft Ans. 1

See above

5 (i) Let $y = 3 + 2x - x^2$

x	y
−2	−5
−1	0
0	3
1	4
2	3
3	0
4	−5



(ii) The roots of the equation $x^2 - 3 = 2x$ are the same as the roots of the equation $3 + 2x - x^2 = 0$ ∴ the roots are the values of x when $y = 0$ or the roots are $x = -1$ and $x = 3$ } Ans. 2 they are the abscissas of the pts. of intersection with the x-axis.

(iii) the function $3 + 2x - x^2$ is always positive for all points on the curve above the x-axis i.e. $3 + 2x - x^2 > 0$ when $-1 < x < 3$ Ans. 3

(iv) the function $3 + 2x - x^2$ is greatest when $x = 1$ + $y_{max} = 4$ Ans. 4

we plot the st. line $y = \frac{x}{2} + 2$ of which two points are: $(-4, 0)$ + $(0, 2)$ we join these points + we get the st. line AB which intersects the curve at $A(2,3)$ + $B(-\frac{1}{2}, 1\frac{1}{4})$
∴ for all values of x between $-\frac{1}{2}$ and 2, the curve of $3 + 2x - x^2$ lies above the st. line $\frac{x}{2} + 2$ ∴ $3 + 2x - x^2 > \frac{x}{2} + 2$ when $-\frac{1}{2} < x < 2$.
(Ans. 5)

(vi) The roots of the equation $3 + 2x - x^2 = \frac{x}{2} + 2$ are the values of x which will make the ordinate of the curve $3 + 2x - x^2$ equal to the ordinate of the st. line $\frac{x}{2} + 2$. But the ordinates of the curve + the line are equal at the pts. of intersection A + B.
∴ the roots are the abscissas of A + B or $x = -\frac{1}{2}$ and $x = 2$
Ans. 6

Shamash Secondary School
Final Examination May 1969

Subject: Algebra
Class: 4th Year, Scientific

Date: 14/5/1969
Time: 8:00 - 11:00 a.m.

Answer all questions:

1. (i) In the expression $x^3+Ax^2+31x+B$, A and B are constant. Find the values of A and B which will make this expression divisible by (x-2) (x-3) and find the remaining factor.
(10 marks)

(ii) The wages of 12 men and 7 boys amount to £9 13s. If 3 men together receive 8s. more than 4 boys, what are the wages of each man and boy ?
(10 marks)

2. (i) Solve the equation $(\log x)^2 = \log x^7 - 10$, finding two values for x.
(10 marks)

(ii) Solve the two simultaneous equations:

$3x^2+xy-2y^2+7=0$ ..... .(1)
$x^2-xy+y^2-7=0$ .....(2) (10 marks)

3. (i) Find the value of x from the following equation without using the tables:

$$(5)(4^{3x-1})(\sqrt{8})^{1-x} = (\sqrt{2})^{x}(\sqrt{50})$$ (10 marks)

(ii) Compute the value of $\sqrt[7]{\frac{(0.5002)^2 \sin^3 14^\circ 25'}{(4.003)^3 \cos^2 15^\circ 27'}}$ (10 marks)

4. (i) If $(b+c)^{-1}$, $(c+a)^{-1}$, $(a+b)^{-1}$ are in arithmetical progression, prove that a^2, b^2, c^2 are also in arithmetical progression.
(10 marks)

(ii) A bouncing tennis ball rebounds each time to a height one half the height of the previous bounce. If it is dropped from a height of 10 ft., show:
(a) that the total distance it has travelled when it hits the ground for the 10th time is equal to $29\frac{123}{128}$ ft. (b) Show also that the total distance it travels before coming to rest is 30 ft.
(10 marks)

(cont'd.p.2)..

(cont'd)..

Algebra 4th Year. Scientific 14/5/1969.

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5. (i) Plot the curve of the function $3+2x-x^2$ for values of x from x=-2 to x=4, choosing one half of an inch for each unit on the axis of x and on the axis of y.

(4 marks)

(ii) From your graph, find the roots of the equation $x^2-3=2x$.

(3 marks)

(iii) Find the values of x for which the function $3+2x-x^2$ is always positive.

(3 marks)

(iv) Find from your diagram the value of x at which the function $3+2x-x^2$ is greatest and state the maximum value.

(3 marks)

(v) By plotting another curve on the same diagram, find the values of x for which $3+2x-x^2 > \frac{x}{2} + 2$. (4 marks)

(vi) From your last diagram, find the roots of the equation $3+2x-x^2 = \frac{x}{2} + 2$.

(3 marks)

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$$3x^2+xy-2y^2+7=0 \quad \ldots\ldots\ldots(1)$$
$$x^2-xy+y^2-7=0 \quad \ldots\ldots\ldots(2)$$
(10 marks)

3. (i) Find the value of x from the following equation without using the tables:

$$(5)(4^{3x-1})(\sqrt{8})^{1-x} = (\sqrt{2})^{x}(\sqrt{50})$$
(10 marks)

(ii) Compute the value of $\sqrt[7]{\dfrac{(0.5002)^2 \sin^3 14^\circ 25'}{(4.003)^3 \cos^2 15^\circ 27'}}$ (10 marks)

4. (i) If $(b+c)^{-1}$, $(c+a)^{-1}$, $(a+b)^{-1}$ are in arithmetical progression, prove that a^2, b^2, c^2 are also in arithmetical progression.
(10 marks)

(ii) A bouncing tennis ball rebounds each time to a height one half the height of the previous bounce. If it is dropped from a height of 10 ft., show:
(a) that the total distance it has travelled when it hits the ground for the 10th time is equal to $29\frac{123}{128}$ ft. (b) Show also that the total distance it travels before coming to rest is 30 ft.
(10 marks)

(cont'd.p.2)..

---

5. (i) Plot the curve of the function $3+2x-x^2$ for values of x from x=-2 to x=4, choosing one half of an inch for each unit on the axis of x and on the axis of y.

(4 marks)

(ii) From your graph, find the roots of the equation $x^2-3=2x$.

(3 marks)

(iii) Find the values of x for which the function $3+2x-x^2$ is always positive.

(3 marks)

(iv) Find from your diagram the value of x at which the function $3+2x-x^2$ is greatest and state the maximum value.

(3 marks)

(v) By plotting another curve on the same diagram, find the values of x for which $3+2x-x^2 > \frac{x}{2} + 2$. (4 marks)

(vi) From your last diagram, find the roots of the equation $3+2x-x^2 = \frac{x}{2} + 2$.

(3 marks)

Solution to the 3rd + 4th Quarter Exam. in Algebra 7/4/1969

(i) The square root of $\left(a^3-\frac{1}{a^3}\right)^2-6\left(a-\frac{1}{a}\right)\left(a^3-\frac{1}{a^3}\right)+9\left(a-\frac{1}{a}\right)^2$ is equal:

$$\sqrt{\left(a^3-\frac{1}{a^3}\right)^2-6\left(a-\frac{1}{a}\right)\left(a^3-\frac{1}{a^3}\right)+9\left(a-\frac{1}{a}\right)^2}=\sqrt{\left[\left(a^3-\frac{1}{a^3}\right)-3\left(a-\frac{1}{a}\right)\right]^2}$$

$$=a^3-\frac{1}{a^3}-3\left(a-\frac{1}{a}\right)=a^3-3a+\frac{3}{a}-\frac{1}{a^3}$$

also $\sqrt[3]{a^3-3a+\frac{3}{a}-\frac{1}{a^3}}=\sqrt[3]{\left(a-\frac{1}{a}\right)^3}=a-\frac{1}{a}$ Ans.

(ii) Prove the identity: $bc(b-c)+ca(c-a)+ab(a-b)=-(b-c)(c-a)(a-b)$

L.H.S. $=bc(b-c)+ac^2-a^2c+a^2b-ab^2$

$=bc(b-c)+a^2(b-c)-a(b^2-c^2)$

$=(b-c)\left[bc+a^2-a(b+c)\right]=(b-c)(bc+a^2-ab-ac)$

$=(b-c)\left[a(a-b)-c(a-b)\right]=(b-c)(a-b)(a-c)$

$=-(b-c)(c-a)(a-b)$ Q.E.D.

2. (i) Solve: $\frac{x-1}{\sqrt{x}-1}=3+\frac{\sqrt{x}+1}{2}$ $\therefore 2(x-1)=6(\sqrt{x}-1)+(\sqrt{x}-1)(\sqrt{x}+1)$

$\therefore 2x-2=6\sqrt{x}-6+x-1$ $\therefore 6\sqrt{x}=x+5$

$\therefore 36x=x^2+10x+25$ $\therefore x^2-26x+25=0$

$\therefore (x-1)(x-25)=0$ $\therefore x=1$

and $x=25$ } Ans.

but $x=1$ does not satisfy the original equation & should be rejected.

Hence $x=25$ Ans.

3 (i) $x=\sqrt[7]{\frac{(0.002001)^3(\sin 16^\circ 23')^2}{(1.003)^5(\tan 41^\circ 16')^2}}$

$\log 0.002001 = \bar{3}.3012$

$\log \sin 16^\circ 23' = \bar{1}.4504$

$\log 1.003 = 0.0012$

$\log \tan 41^\circ 16' = \bar{1}.9433$

$3\log 0.002001 = \bar{9}.9036$

$2\log \sin 16^\circ 23' = \bar{2}.9008$

$\log$ Num. $= \bar{10}.8044$

$\log$ Den. $= \bar{1}.8926$

$7\log x = \bar{10}.9118$

$\log x = \bar{2}.7017$ 0.05031

$x = 0.07975$

or $x = 7.975 \times 10^{-2}$ } Ans.

$5\log 1.003 = 0.0060$

$2\log \tan 41^\circ 16' = \bar{1}.8866$

$\log$ Den $= \bar{1}.8926$

$$\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} = \frac{\left(\sqrt{1+x}+\sqrt{1-x}\right)^2}{1+x-(1-x)} = \frac{1+x+1-x+2\sqrt{1-x^2}}{2x} = \frac{2+2\sqrt{1-x^2}}{2x} =$$

$$= \frac{2(1+\sqrt{1-x^2})}{2x} = \frac{1+\sqrt{1-x^2}}{x} = \frac{1+\sqrt{1-\frac{4b^2}{(b^2+1)^2}}}{\frac{2b}{b^2+1}} = \frac{1+\frac{\sqrt{(b^2+1)^2-4b^2}}{b^2+1}}{\frac{2b}{b^2+1}} =$$

$$= \frac{b^2+1+\sqrt{b^4+2b^2+1-4b^2}}{2b} = \frac{b^2+1+\sqrt{(b^2-1)^2}}{2b} = \frac{b^2+1+b^2-1}{2b} = \frac{2b^2}{2b} = b \quad \text{Ans.}$$

3(ii) Solve for x:

$$2(\log x)^2 - 5(\log x) + 2 = 0 \quad \therefore [2(\log x)-1][\log x - 2] = 0$$

$\therefore 2\log x = 1$ or $\log x = \frac{1}{2}$ $\therefore x = 10^{\frac{1}{2}}$ or $x = \sqrt{10} = 3.162$ correct to 3 dec. pl.

or $\log x = 2$ or $x = 10^2$ or $x = 100$ } Ans.

4.(i) $a = 3$, $d = 6$ } Prove $S_{2n} = 4S_n$

$S_{2n} = \frac{2n}{2}\{2a+(2n-1)d\} = n\{6+(2n-1)(6)\} = n\{6+12n-6\} = 12n^2$

$S_n = \frac{n}{2}\{6+(n-1)(6)\} = \frac{n}{2}\{6n\} = 3n^2$

but $12n^2 = 4(3n^2)$ $\therefore S_{2n} = 4S_n$ Q.E.D.

(ii) $\frac{l_3}{l_6} = \frac{11}{26}$ and $S_4 = 34$, $S_8 = ?$

$\therefore \frac{a+2d}{a+5d} = \frac{11}{26}$ or $26a+52d = 11a+55d$ or $15a-3d = 0$ ..... ①

also $34 = \frac{4}{2}\{2a+3d\}$ or $34 = 2(2a+3d)$ or $17 = 2a+3d$ ..... ②

$\therefore$ { $15a-3d=0$ ..... ① ; $2a+3d=17$ ..... ② } or ~~{ $20a+7d=9$; $20a+30d=170$ } $\therefore 23d = 170$ $\therefore d \neq$~~

$17a = 17$ $\therefore a = 1$ $\therefore 15 = 3d$ $\therefore d = 5$ $\therefore$ $a=1$, $d=5$ } Ans. 1

+ the progression is 1, 6, 11, 16, ...

$S_8 = \frac{8}{2}\{2\times1+7\times5\}$ or $S_8 = 4(2+35) = 4\times37 = 148$ Ans. 2

Shamash Secondary School
3rd Quarter + 4th Quarter Examination

Subject : Algebra
Class : 4th Year, Scientific

Date : 7/4/1969
Time: 8:30 – 10:30 a.m.

Answer all questions:

1. (i) By first taking the square root and then the cube root, find the sixth root of:

$$\left(a^3-\frac{1}{a^3}\right)^2-6\left(a-\frac{1}{a}\right)\left(a^3-\frac{1}{a^3}\right)+9\left(a-\frac{1}{a}\right)^2 \qquad \text{(12 marks)}$$

(ii) Prove that the left hand side is always equal to the right hand side in the following equation:

$$bc(b-c)+ca(c-a)+ab(a-b)=-(b-c)(c-a)(a-b) \qquad \text{(13 marks)}$$

2. (i) solve the equation : $\frac{x-1}{\sqrt{x}-1}=3+\frac{\sqrt{x}+1}{2}$ (12 marks)

(ii) Rationalise the denominator and then find the value of :

$$\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}, \text{ when } x=\frac{2b}{b^2+1} \qquad \text{(13 marks)}$$

3. (i) Compute : $\sqrt[7]{\frac{(6.002001)^3(\sin 16^\circ 23')^2}{(1.003)^5(\tan 41^\circ 16')^2}}$ (12 marks)

(ii) Solve the following equation for x:

$$2(\log x)^2-5(\log x)+2=0 \qquad \text{(13 marks)}$$

4. (i) In an Arithmetic Progression the first term is 3 and the common difference is 6. Show that the sum of $2n$ terms is always equal to four times the sum of n terms. (12 marks)

(ii) In an A.P. the ratio of the 3rd term to the 6th term is 11:26 and the sum of the first 4 terms is 34. Find the progression and the sum of the first 8 terms. (13 marks)

SHAMASH SECONDARY SCHOOL

3rd & 4th quarter Examination.

Subject : Algebra
Class : 4th Year Scientific

Date : 7/4/1969.
Time : 8:30-10:30 a.m.

Answer all Questions :

1. (i) By first taking the square root and then the cube root, find the sixth root of :

$$\left(a^3 - \frac{1}{a^3}\right)^2 - 6\left(a - \frac{1}{a}\right)\left(a^3 - \frac{1}{a^3}\right) + 9\left(a - \frac{1}{a}\right)^2. \qquad \text{(12 marks).}$$

(ii) Prove that the left-hand side is always equal to the right-hand side in the following equation :

$$bc(b - c) + ca(c - a) + ab(a - b) = -(b - c)(c - a)(a - b).$$

(13 marks).

2. (i) Solve the equation : $\dfrac{x - 1}{\sqrt{x} - 1} = 3 + \dfrac{\sqrt{x} + 1}{2}$. (12 marks).

(ii) Rationalise the denominator and then find the value of :

$$\frac{\sqrt{1 + x} + \sqrt{1 - x}}{\sqrt{1 + x} - \sqrt{1 - x}}, \quad \text{when } x = \frac{2b}{b^2 + 1}.$$

(13 marks).

3. (i) Compute by logarithms : $\sqrt[7]{\dfrac{(0.002001)^3(\sin 16^\circ 23')^2}{(1.003)^5(\tan 41^\circ 16')^2}}$ (12 marks).

(ii) Solve the following equation for x :

$$2(\log x)^2 - 5(\log x) + 2 = 0.$$

(13 marks).

4. (i) In an Arithmetic Progression the first term is 3 and the common difference is 6. Show that the sum of 2n terms is always equal to four times the sum of n terms. (12 marks).

(ii) In an A. P. the ratio of the 3rd term to the 6th term is 11:26 and the sum of the first 4 terms is 34. Find the progression and the sum of the first 8 terms. (13 marks).

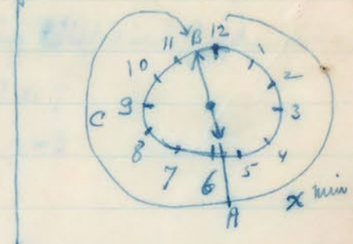
Solutions to Mid-year Exam., in Algebra
4th Secondary year, February, 1969.

1. Let the time now be x minutes after 5 o'clock.

Then $x = 25 + \frac{x}{12} + 30$ $(ACB = 30)$

$\therefore x - \frac{x}{12} = 55$ $\therefore \frac{11x}{12} = 55$ $\therefore x = 60$ minutes after 5

$\therefore$ the time now is exactly 6 o'clock Ans.



2. (i) solve $6x^3+19x^2+x-6=0$

By trial + error we discover that $x=-3$ satisfies the equation. Hence by the remainder + factor theorems, $(x+3)$ is a factor. Factoring, we get

$(x+3)(6x^2+x-2)=0$ or $(x+3)(3x+2)(2x-1)=0$

$\therefore$ $x=-3$, $x=-\frac{2}{3}$, $x=\frac{1}{2}$ } Ans.

(ii) $\frac{3x-7}{x-2} + \frac{2x-5}{x-3} = \frac{3x+7}{x+2} + \frac{2x+5}{x+3}$

$\therefore \frac{3(x-2)-1}{x-2} + \frac{2(x-3)+1}{x-3} = \frac{3(x+2)+1}{x+2} + \frac{2(x+3)-1}{x+3}$

$\therefore 3 - \frac{1}{x-2} + 2 + \frac{1}{x-3} = 3 + \frac{1}{x+2} + 2 - \frac{1}{x+3}$

$\frac{1}{x-3} - \frac{1}{x-2} = \frac{1}{x+2} - \frac{1}{x+3}$ $\therefore \frac{x-2-x+3}{(x-2)(x-3)} = \frac{x+3-x-2}{(x+2)(x+3)}$

$\therefore \frac{1}{(x-2)(x-3)} = \frac{1}{(x+2)(x+3)}$ $\therefore (x+2)(x+3) = (x-2)(x-3)$

$\therefore x^2+5x+6 = x^2-5x+6$ $\therefore 10x=0$ $\therefore x=0$ Ans.

3. (i) $A(x^2-2x)+B(x+4)+C = 3x^2+x+25$

$\therefore Ax^2-2Ax+Bx+4B+C = 3x^2+x+25$

$\therefore Ax^2+(B-2A)x+4B+C = 3x^2+x+25$. Equating coefficients of like terms,

$\therefore A=3$, $B-2A=1$ $\quad \therefore B-6=1$ or $B=7$

$4B+C=25$ $\quad \therefore 4\times7+C=25$ or $C=-3$

$\therefore A=3$, $B=7$ and $C=-3$ Ans.

Another method:

In the original identity, let $x=0$, then $4B+C=25$ ... ①

again let $x=2$, then $6B+C=12+2+25=39$ ... ②

$6B+C=39$ ... ②

$4B+C=25$ ... ① | subtract $2B=14$ $\therefore B=7$ and $C=-3$

Now let $x=1$, then $A(1-2)+B(1+4)+C=3+1+25$ or

$-A+5\times7-3=29$

$\therefore A=32-29$ $\therefore A=3$, $B=7$ and $C=-3$ Ans.

(ii) Solve : $x^2+xy+2y^2=8$ ..... ①

$2x^2-2xy-3y^2=1$ ..... ②

Let $y=mx$ $\therefore x^2+mx^2+2m^2x^2=8$ from ① ..... (1.a)

$2x^2-2mx^2-3m^2x^2=1$ from ② ..... (2.a)

$\therefore \left.\begin{matrix} x^2(1+m+2m^2)=8 \\ x^2(2-2m-3m^2)=1 \end{matrix}\right\}$ dividing, $\frac{1+m+2m^2}{2-2m-3m^2}=\frac{8}{1}$

$\therefore 1+m+2m^2=16-16m-24m^2$ $\therefore 26m^2+17m-15=0$

$\therefore (13m+15)(2m-1)=0$ $\therefore m=\frac{1}{2}$ or $m=-\frac{15}{13}$

when $m=\frac{1}{2}$, from (1.a), we get: $x^2+\frac{x^2}{2}+\frac{x^2}{2}=8$ or $2x^2=8$

$\therefore x^2=4$ $\therefore x=\pm2$ when $x=2$, $y=mx=\frac{1}{2}\times2=1$

and when $x=-2$, $y=\frac{1}{2}(-2)=-1$

$\therefore \left.\begin{matrix} x=2 \\ y=1 \end{matrix}\right\}$ Ans. 1 $\quad \left.\begin{matrix} x=-2 \\ y=-1 \end{matrix}\right\}$ Ans. 2

when $m=-\frac{15}{13}$ then $x^2\left(1-\frac{15}{13}+2\times\frac{225}{169}\right)=8$ $\therefore x^2\left(\frac{169-13\times15+450}{169}\right)=8$

$\therefore x^2\left(\frac{169-195+450}{169}\right)=8$ $\therefore x^2\left(\frac{424}{169}\right)=8$ $\therefore x^2=\frac{8\times169}{424}=\frac{169}{53}$

$\therefore x=\pm\frac{13}{\sqrt{53}}$ when $x=\frac{13}{\sqrt{53}}$, $y=-\frac{15}{13}\times\frac{13}{\sqrt{53}}=-\frac{15}{\sqrt{53}}$ and when $x=-\frac{13}{\sqrt{53}}$, $y=\left(-\frac{15}{13}\right)\left(-\frac{13}{\sqrt{53}}\right)=\frac{15}{\sqrt{53}}$

$\therefore \left.\begin{matrix} x=\frac{13}{\sqrt{53}} \\ y=-\frac{15}{\sqrt{53}} \end{matrix}\right\}$ Ans. 3 $\quad \left.\begin{matrix} x=-\frac{13}{\sqrt{53}} \\ y=\frac{15}{\sqrt{53}} \end{matrix}\right\}$ Ans. 4

An alternative method

$x^2+xy+2y^2=8$ ..... ①

$2x^2-2xy-5y^2=11$ ..... ②

multiply Eq (2) by 8 :

$16x^2-16xy-24y^2=8$ ..... ③

Subtract $x^2+xy+2y^2=8$ ..... ①

$15x^2-17xy-26y^2=0$ ..... ④

$\therefore (x-2y)(15x+13y)=0$

$\therefore y=\frac{1}{2}x$ or $y=-\frac{15}{13}x$

When $y=\frac{x}{2}$ from Eq. ① : $x^2+x(\frac{x}{2})+2(\frac{x}{2})^2=8$ or $x^2+\frac{x^2}{2}+\frac{x^2}{2}=8$

or $2x^2=8$ $\therefore x^2=4$ $\therefore x=\pm 2$ $\therefore y=\frac{x}{2}=\frac{\pm 2}{2}=\pm 1$

$\therefore x=2$ $y=1$ Ans. 1 $\quad x=-2$ $y=-1$ Ans. 2

when $y=-\frac{15}{13}x$ from Eq. ① : $x^2+x(-\frac{15}{13}x)+2(-\frac{15}{13}x)^2=8$ or

$x^2-\frac{15}{13}x^2+\frac{450}{169}x^2=8$ $\therefore 169x^2-15\times 13x^2+450x^2=8\times 169$

$\therefore 169x^2-195x^2+450x^2=1352$ $\therefore 424x^2=1352$ $\therefore x^2=\frac{1352}{424}$

$\therefore x^2=\frac{169}{53}$ $\therefore x=\pm\frac{13}{\sqrt{53}}$

when $x=\frac{13}{\sqrt{53}}$ $y=-\frac{15}{13}x$ or $y=-\frac{15}{13}(\frac{13}{\sqrt{53}})=-\frac{15}{\sqrt{53}}$

and when $x=-\frac{13}{\sqrt{53}}$ $y=-\frac{15}{13}x$ or $y=-\frac{15}{13}(-\frac{13}{\sqrt{53}})=\frac{15}{\sqrt{53}}$

$\therefore x=\frac{13}{\sqrt{53}}$ $y=-\frac{15}{\sqrt{53}}$ Ans. 3 $\quad x=-\frac{13}{\sqrt{53}}$ $y=\frac{15}{\sqrt{53}}$ Ans. 4

4(i) $x^3+y^3+\frac{1}{x^3}+\frac{1}{y^3} = \left(x^3+\frac{1}{x^3}\right)+\left(y^3+\frac{1}{y^3}\right) = \left[\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)\right]+\left[\left(y+\frac{1}{y}\right)^3-3\left(y+\frac{1}{y}\right)\right]$

$= a^3-3a+b^3-3b = a^3+b^3-3(a+b)$

$= (a+b)(a^2-ab+b^2-3)$ Ans. 1

$= (1+2)(1^2-1\times2+2^2-3)$

$= (3)(1-2+4-3) = 3\times \text{zero} = 0$ Ans. 2

(ii) $a^3+a-8b^3-2b+c+6abc+c^3 = a^3-8b^3+c^3+6abc+a-2b+c$

$= \left[a^3+(-2b)^3+c^3-3a(-2b)c\right]+\left[a-2b+c\right]$

$= (a-2b+c)(a^2+4b^2+c^2+2ab-ac+2bc)+(a-2b+c)$

$= (a-2b+c)\left[(a^2+4b^2+c^2+2ab-ac+2bc)+1\right]$

$= (a-2b+c)(a^2+4b^2+c^2+2ab-ac+2bc+1)$ Ans.

5. after the first replacement, there are
$\frac{x}{2}$ gall. of Brandy in cask P and
$(50-\frac{x}{2})$ gall. " " " " Q

At the beginning of the 2nd operation:
$\left(\frac{x^2}{200}\right)$ gall. of Brandy are removed from cask P and

$\frac{x(50-\frac{x}{2})}{50}$ gall. " " " " " Q

100 gall. Water	50 gall. Brandy
Mixture 100	Brandy $\frac{x}{2}$ gall.
x	$\frac{x}{2}$ gall.

$? = x\cdot\frac{x}{2}\cdot\frac{1}{100} = \frac{x^2}{200}$ gall.

50 gall.	$(50-\frac{x}{2})$ gall.
x	?

$? = \frac{x(50-\frac{x}{2})}{50}$

$\frac{\frac{x^2}{200}+\frac{x(50-\frac{x}{2})}{50}}{2}$ gall. of Brandy are deposited in P after 2nd replacement.

$\therefore \left(\frac{x}{2}-\frac{x^2}{200}\right)+\frac{\frac{x^2}{200}+\frac{x(50-\frac{x}{2})}{50}}{2} = 17$ or

$\frac{100x-x^2}{200}+\frac{\frac{x^2+4x(50-\frac{x}{2})}{200}}{2} = 17$ $\therefore \frac{100x-x^2}{200}+\frac{\frac{x^2+200x-2x^2}{200}}{2} = 17$

$\therefore \frac{100x-x^2}{200}+\frac{200x-x^2}{400} = 17$ $\therefore 200x-2x^2+200x-x^2 = 6800$

$\therefore 3x^2-400x+6800=0$ $\therefore (3x-340)(x-20)=0$

$\therefore x = \frac{340}{3} = 113\frac{1}{3}$ inadmissible

$x = 20$ gallons Ans.

6. Speed of train A = 22.5 mi/h = $22.5 \times \frac{22}{15}$ ft/sec = $\frac{45}{2} \times \frac{22}{15}$ = 33 ft/sec.
Speed " " B = 15 mi/h = $15 \times \frac{22}{15}$ ft/sec = 22 ft/sec.

(i) When the two trains are travelling in opposite directions (see Fig. I), points C and D are separating at the rate of (33+22) ft/sec = 55 ft/sec.
When the rear cars A and B just clear away from each other, (see Fig. II) points C and D have already separated by a distance = (240+200) ft = 440 ft.
∴ time taken = $\frac{\text{total distance}}{\text{rate of separation}} = \frac{440}{55} = 8$ sec. Ans. 1

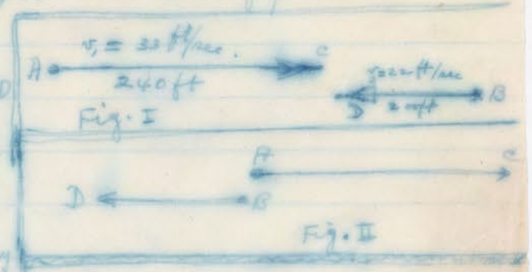


Fig. I

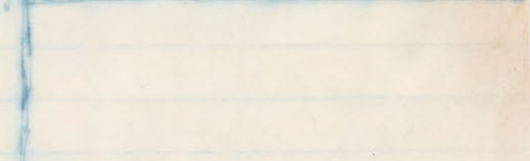


Fig. II

(ii) When the two trains are travelling in the same direction, (See Fig. III), points C and D are separating at the rate of (33−22) ft/sec = 11 ft/sec.
When the rear car A of the faster train and the front car D of the slower train just clear away from each other, (see Fig. IV), points C and D have already separated by a distance of 240 ft which is the length of the faster train.

∴ time taken = $\frac{\text{distance}}{\text{rate of separation}} = \frac{240}{11} = 21\frac{9}{11}$ sec. Ans. 2

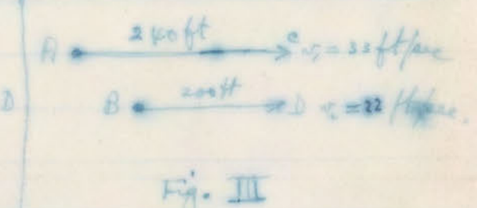


Fig. III

Fig. IV

SHAMASH SECONDARY SCHOOL

Mid-Year Examination, February, 1969.

Subject : Algebra
Class : 4th Year, Secondary

Date : 17/2/1969.
Time : 8:30-11:30 a.m.

Five questions only are to be attempted.

1. The time now is x minutes after five and the two hands of the watch stand in a straight line on opposite sides of the centre of the dial of the watch. Find x and state, in words, the correct time. (20 marks)

2. (i) Find the value of x from the following equation :

$$6x^3 + 19x^2 + x - 6 = 0.$$ (10 marks)

(ii) Solve the following equation, using the shortest possible method, by first reducing each fraction to a simpler form :

$$\frac{3x-7}{x-2} + \frac{2x-5}{x-3} = \frac{3x+7}{x+2} + \frac{2x+5}{x+3}.$$ (10 marks)

3. (i) In the following equation, A, B, and C are constants and the equation is true for all values of x. Find the values of A, B and C.

$$A(x^2 - 2x) + B(x + 4) + C = 3x^2 + x + 25.$$ (10 marks)

(ii) Solve the following equations simultaneously :

$$x^2 + xy + 2y^2 = 8 \quad \ldots\ldots\ldots\ldots (1)$$
$$2x^2 - 2xy - 3y^2 = 1 \quad \ldots\ldots\ldots\ldots (2)$$ (10 marks)

4. (i) If $x + \frac{1}{x} = a$ and $y + \frac{1}{y} = b$, find the value of the expression $(x^3 + y^3 + \frac{1}{x^3} + \frac{1}{y^3})$ in terms of "a" and "b". Hence or otherwise find the value of $(x^3 + y^3 + \frac{1}{x^3} + \frac{1}{y^3})$ if $a = 1$ and $b = 2$. (10 marks)

(ii) Resolve the expression $a^3 + a - 8b^3 - 2b + c + 6abc + c^3$ into two factors one of which is $(a - 2b + c)$. (10 marks)

5. A cask P is filled with 100 gallons of water, and a cask Q with 50 gallons of brandy; x gallons are drawn from each cask, mixed and replaced; and the same operation is repeated. Find x when there are 17 gallons of brandy in P after the second replacement. (20 marks)

6. Two trains A and B are travelling on two railway tracks which are parallel to each other. Train A is 240 ft long and it is travelling at 22.5 miles per hour. Train B is 200 ft long and is travelling at the rate 15 miles per hour. Find the length of time in seconds from the instant when the heads of the front cars of the two trains are together, to the instant when the

P. T. O.

Mid-Year Exam. Cont., in Algebra ; 4th Year, Secondary, 17/2/1969.

two trains just clear away from each other in the two cases :

(i) When the two trains are travelling in opposite directions.

(ii) when the two trains are travelling in the same direction.

(20 marks).

Solution to Algebra Exam. (2nd part) : 17/2/1969 page 1

1. $2x^3+Ax^2+Bx-4$ | $x-2$

$2x^3-4x^2$ | $2x^2+(4+A)x+(8+2A+B)$ | $x+2$

$(4+A)x^2+Bx-4$ | $2x^2+4x$ | $2x+A$

$(4+A)x^2-2(4+A)x$

$[2(4+A)+B]x-4$ | $[(4+A)-4]x+8+2A+B$

$(8+2A+B)x-2(8+2A+B)$ | or $Ax+2A+B+8$

$2(8+2A+B)-4=0$ | $Ax+2A$

$\therefore 4+2A+2B=-12$ | $B+8=0$ ---(2)

or $2A+B=-6$ ----(1) | $\therefore B=-8$ from (1) $2A-8=-6 \therefore A=1$

$\therefore A=1, B=-8$ } Ans.1

$\therefore$ the remaining factor is the last quotient, namely : $2x+A$ or $2x+1$ Ans.2

$[x^2-4=(x-2)(x+2)]$

An alternative method: the factors of x^2-4 are $(x-2)$ & $(x+2)$, By the principle of remainder, when $x=2$ the expression $2x^3+Ax^2+Bx-4$ becomes zero

$\therefore 2\times8+4A+2B-4=0$ or $4A+2B=-12$ or $2A+B=-6$ ---- (1)

also when $x=-2$, then $2(-8)+4A-2B-4=0$ or $4A-2B=20 \therefore 2A-B=10$ ---- (2)

now $2A+B=-6$ ---(1) $\therefore 4A=4 \therefore A=1$

$2A-B=10$ ---(2) $\therefore B=-8$ } Ans. 1

$\therefore 2x^3+x^2-8x-4=(x^2-4)(2x+1) \therefore (2x+1)$ is the remaining factor Ans. 2

2. $\frac{9}{4}x^6-3x^5+4x^4-3x^3+\frac{5}{3}x^2-\frac{2}{3}x+\frac{1}{9}$ (arranging according to descending powers of x)

$= \frac{9}{4}x^6-3x^5+4x^4-3x^3+\frac{5}{3}x^2-\frac{2}{3}x+\frac{1}{9}$ | $\frac{3}{2}x^3-x^2+x-\frac{1}{3}$

$\frac{9}{4}x^6$

$3x^3-x^2$ | $-3x^5+x^4-3x^3$

$x \quad -x^2$ | $-3x^5+x^4$

$3x^3-2x^2+x$ | $+\frac{5}{3}x^4-2x^3+\frac{5}{3}x^2$

$x \quad +\frac{1}{3}x$ | $+\frac{5}{3}x^4-2x^3+\frac{5}{3}x^2$

$3x^3-2x^2+2x-\frac{1}{3}$ | $-x^3+\frac{5}{3}x^2-\frac{2}{3}x+\frac{1}{9}$

$-\frac{1}{3}$ | $-x^3+\frac{5}{3}x^2-\frac{2}{3}x+\frac{1}{9}$

$\therefore$ the square root is :

$\frac{3}{2}x^3-x^2+x-\frac{1}{3}$ Ans.

(20 marks)

3.

$=1$

5. Let the certain sum of money be £X = 20X shillings

∴ 1 lb Coffee costs $\frac{20X}{A}$ shillings

∴ 1 lb " " sold for $\left(\frac{20X}{A}+C\right)$ shillings

(A−B) lbs of Coffee sold for $(A-B)\left(\frac{20X}{A}+C\right)$ shillings

∴ $20X=(A-B)\left(\frac{20X}{A}+C\right)$ or $20AX=(A-B)(20X+AC)$

or $20AX=20X(A-B)+AC(A-B)$ or

or $20AX-20(A-B)X=AC(A-B)$

∴ $20X\left[A-(A-B)\right]=AC(A-B)$ or

$20BX=AC(A-B)$ ∴ $X=£\frac{AC(A-B)}{20B}$ Ans.

Make-up Exam. 2nd Quarter

17/1/1969

I (i) Find the value of $x^2 - \frac{1}{y}$ when $x = -\frac{1}{2}$, $y = -3$ (6 marks)

(ii) Solve the equation $\frac{x}{2} + \frac{x-1}{3} = 7$ (7 marks)

(iii) Find x if $2x + y = 4$ and $3y + 4y = 6x$ (7 marks)

II (i) If $t = \sqrt[3]{\frac{x^2 + 4z}{2yz}}$, find z in terms of x, y and t. (10 marks)

(ii) If $F = av - \frac{b}{v^2}$ and if $F = 4$ when $v = 5$ and $F = 36$ when $v = 10$, find the values of "a" and "b" and the value of F when $v = 20$ (10 marks)

III a man can cycle at x m.p.h. in still air. His speed increases y m.p.h. when he cycles with the wind, and decreases y m.p.h. when he cycles against the wind. The difference in his time to cycle one mile with the wind and one mile against the wind is z hours. Find a formula for z in terms of x and y, and find x if $y = 2$, $z = \frac{4}{5}$. (20 marks)

IV. In how many days will a horses eat $\frac{1}{n}$ th of the corn of a field the whole of which can be eaten by "b" horses in "c" days. (20 marks)

V. Find the square root of:

$$16x^4 + \frac{16}{3}x^3y + 8x^2 + \frac{4}{9}y^2 + \frac{4}{3}y + 1$$

showing your steps neatly. (20 marks)

SHAMASH SECONDARY SCHOOL

Subject: Algebra
Class : 4th Year Secondary

Date: 7/1/1969
Time: 10:15-11:45

Attempt all questions:

1. The expression $2x^3 + Ax^2 + Bx - 4$ is exactly divisible by x^2-4. Find the values of A and B and find the remaining factor.

(20 marks)

2. Find the square root of:

$$4x^4-3x^5-3x^3+\frac{9}{4}x^6+\frac{5}{3}x^2-\frac{2}{3}x+\frac{1}{9}.$$

(20 marks)

3. Which of the following equations is always true, which is sometimes true and which is never true? Find the values of x which satisfy the equation which is sometimes true.

(a) $4(x^2-1) + 2(x + 3) = 2 + 2x(1 + 2x)$
(b) $x(6x + 1) = 2x + 1$
(c) $x(x + 2) = 2(x -2)$

(20 marks)

4. (i) Solve simultaneously the following equations:

$$\frac{1}{x}+\frac{1}{y}+\frac{3}{z} = 2\tfrac{1}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\frac{2}{x}+\frac{4}{y}-\frac{6}{z} = 2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\frac{3}{x}-\frac{5}{y}+\frac{7}{z} = 2\frac{5}{6} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

(ii) Simplify the following expression to simplest form:

$$\frac{\frac{x}{y}+\frac{y}{x}-1}{\frac{x^2}{y^2}+\frac{x}{y}+1} \cdot \frac{1+\frac{y}{x}}{x-y} \div \frac{1+\frac{y^3}{x^3}}{\frac{x^2}{y}-\frac{y^2}{x}}$$

(20 marks)

5. A man bought "A" lbs of coffee for a certain sum of money. He kept "B" lbs to himself and sold the remainder at "C" shillings a pound more than he paid for it. He found that he received for this portion an amount equal to the original sum of money which he paid for the whole. Find the original sum of money which he paid for the whole. (20 marks)

SHAMASH SECONDARY SCHOOL

Subject: Algebra
Class : 4th Year Secondary

Date: 7/1/1969
Time: 10:15-11:45

Attempt all questions:

1. The expression $2x^3 + Ax^2 + Bx - 4$ is exactly divisible by $x^2 - 4$. Find thevalues of A and B and find the remaining factor.

(20 marks)

2. Find the square root of:

$$4x^4 - 3x^5 - 3x^3 + \frac{9}{4}x^6 + \frac{5}{3}x^2 - \frac{2}{3}x + \frac{1}{9}.$$

(20 marks)

3. Which of the following equations is always true, which is sometimes true and which is never true? Find the values of x which satisfy the equation which is sometimes true.

(a) $4(x^2-1) + 2(x + 3) = 2 + 2x(1 + 2x)$
(b) $x(6x + 1) = 2x + 1$
(c) $x(x + 2) = 2(x - 2)$

(20 marks)

4. (i) Solve simultaneously the following equations:

$$\frac{1}{x} + \frac{1}{y} + \frac{3}{z} = 2\frac{1}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\frac{2}{X} + \frac{4}{y} - \frac{6}{z} = 2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\frac{3}{x} - \frac{5}{y} + \frac{7}{z} = 2\frac{5}{6} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

(ii) Simplify the following expression to simplest form:

$$\frac{\frac{x}{y} + \frac{y}{x} - 1}{\frac{x^2}{y^2} + \frac{x}{y} + 1} \cdot \frac{1 + \frac{y}{x}}{x - y} \div \frac{1 + \frac{y^3}{x^3}}{\frac{x^2}{y} - \frac{y^2}{x}}$$

(20 marks)

5. A man bought "A" lbs of coffee for a certain sum of money. He kept"B" lbs to himself and sold the remainder at "C" shillings a pound more than he paid for it. He found that he received for this portion an amount equal to the original sum of money which he paid for the whole. Find the original sum of money which he paid for the whole.

(20 marks)

SHAMASH SECONDARY SCHOOL

الرقم:
الاسم:

Monthly Examination, November 1968

Subject: General Mathematics — Date: 18/11/1968.

Class : 4th Year Secondary — Time: 8:30 - 10:00 a.m.

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1. Give the English Equivalent of the following, filling the blanks in this sheet and hand it over with your examination book.

Numerals = figures	١ـ ارقام	1 mark
Digits	٢ـ مراتب	"
Subtraction	٣ـ الطرح	"
Factors	٤ـ العوامل	"
The index or exponent of the power	٥ـ اس القوة	"
Multiple	٦ـ مضاعف	"
Consecutive even numbers	٧ـ اعداد زوجية متتالية	"
" odd "	٨ـ اعداد فردية متتالية	"
The integral part of a number	٩ـ الجزء الصحيح من العدد	"
Prime numbers	١٠ـ اعداد اولية	"
The least common Denominator	١١ـ المقام المشترك الاصغر	"
An improper fraction	١٢ـ كسر لفظي	"
The reciprocal of a number	١٣ـ مقلوب العدد	"
Terminating decimals	١٤ـ الكسور العشرية المنتهية	"
Recurring or Repeating decimals	١٥ـ الكسور العشرية الدورية	"
The percentage error	١٦ـ الخطأ المئوي	"
Ratio + Proportion	١٧ـ النسبة والتناسب	"
The mean proportional between two numbers	١٨ـ الوسط المتناسب بين عددين	"
The Dividend	١٩ـ ربح المساهم (ربح حامل الاسهم)	"
Axiom	٢٠ـ البديهية	"
Postulate	٢١ـ الموضوعة	"
an acute angle	٢٢ـ زاوية حادة	"
an obtuse "	٢٣ـ زاوية منفرجة	"
a Reflex "	٢٤ـ زاوية منعكسة	"
a segment of a circle	٢٥ـ قطعة دائرة	"
a sector " " "	٢٦ـ قطاع دائرة	"
The Data	٢٧ـ المعاليم	"
The unknowns	٢٨ـ المجاهيل	"
Two Complementary angles	٢٩ـ زاويتان متتامتان	"
" Supplementary "	٣٠ـ زاويتان متكاملتان	"
an equilateral polygon	٣١ـ مضلع متساوى الاضلاع	"
an isosceles triangle	٣٢ـ مثلث متساوى الساقين	"
The Chords	٣٣ـ المعين	"
The arcs	٣٤ـ المحل الهندسي	"
The secant to a circle	٣٥ـ المستقيم القاطع للدائرة	"
The removal + insertion of Brackets	٣٦ـ ازالة وادخال الاقواس	"
Transposemptom one side to the other	٣٧ـ نقل حدود المعادلة من جهة الى الجهة الاخرى	"
Identity	٣٨ـ متطابقة	"
Inequality	٣٩ـ متباينة	"

ـ يتبع ـ

الرقم:
الاسم:

٤٠ — مقدار جبرى متجانس	A homogeneous algebraic expression	1 mark
٤١ — درجة المقدار الجبرى	The degree or the dimension of an algebraic expression	"
٤٢ — المعامل الحرفي	The literal coefficient	"
٤٣ — مقدار جبرى من الدرجة الثانية	An algebraic expression of the second degree or a quadratic expression	"
٤٤ — ان حدى الكسر هما بسطه ومقامه	The two terms of a fraction are its numerator and denominator	2 "

٤٥ — في كل عملية قسمة يوجد مقسوم ومقسوم عليه وناتج قسمة وفي بعض الحالات باق للقسمة . 5 marks
In every process of division there is a dividend, a divisor, a quotient + in some cases a remainder

٤٦ — ان الاعمدة المنصفة لاضلاع مثلث تلتقي في مركز الدائرة المرسومة . ..... ½
The perpendicular bisectors of the sides of a triangle meet at the centre of the circumscribed circle.

٤٧ — ان الخطوط المتوسطة في المثلث تلتقي في نقطة واحدة تقسم كلا منها الى ثلثين من جهة الر... ½
وثلث من جهة القاعدة . وتسمى هذه النقطة مركز ثقل المثلث.
The medians of a triangle meet at a point which divides each of them two thirds from the vertex and one third from the base. This point is called the centroid of the triangle.

٤٨ — نقيس طول مستقيم فنجد انه يساوى ٦١٫٥سم. ثم نجد فيما بعد ان طوله المضبوط ٦٠سم. ½
وفي هذه الحالة نقول ان الخطأ المطلق هو . ..... 1.5 cms
والخطأ النسبي هو . ..... والخطأ المئوى هو ..... 1/40 - 2.5%
We measure the length of a st. line + we find that it is equal 61.5 cms. Its then found that its exact length is 60 cms. In this case we say that the absolute error is 1.5 cms, the relative error is 1/40 and the percentage error is 2.5%

٤٩ — ان قيمة المقدار ٥٣٠٩٧٢ لاقرب اربعة ارقام معنوية هي .. 5 marks
The value of 530972 correct to 4 significant figures is 5310,00

٥٠ — ان المعادلة ٣س — ٢س ص + ص = ٥ع — ٣ع هي معادلة من الدرجة ... في ... مجاهيل.
The equation $3x - 2xy + y = 5z - 3z$ is a quadratic equation in three unknowns.

(75 marks)

(II) Fill in the blanks in the following equations:-

(2.5 marks)

1. one furlong = (10) chains = (1/8) mile
2. one chain = (22) yards = (100) links
3. one statute mile = (1760) yds. = (5280) ft.
4. one nautical mile = (6080) ft.
5. one sq. chain = (484) sq. yds.
6. one acre = (10) sq. ch. = (4840) sq. yds.
7. one gallon = (8) pints
8. one bushel = (8) gallons = (4) pecks
9. one English ton = (2240) lbs. = (1016) kilograms
10. one English ton = (20) cwt. = (80) qr. = (160) stones.

1 quarter = 1/4 of one cwt = 28 lbs = 2 stones
1 stone = 14 lbs

(25 marks).

SHAMASH SECONDARY SCHOOL

الرقم:
الاسم:

Monthly Examination, November 1968

Subject: General Mathematics — Date: 18/11/1968.

Class : 4th Year Secondary — Time: 8:30 - 10:00 a.m.

--- --- --- ---

1. Give the English Equivalent of the followinh, filling the blanks in this sheet and hand it over with your examination book.

١ــ ارقام
٢ــ مراتب
٣ــ الطرح
٤ــ العوامل
٥ــ اس القوة
٦ــ مضاعف
٧ــ اعداد زوجية متتالية
٨ــ اعداد فردية متتالية
٩ــ الجزء الصحيح من العدد
١٠ــ اعداد اولية
١١ــ المقام المشترك الاصغر
١٢ــ كسر لفظي
١٣ــ مقلوب العدد
١٤ــ الكسور العشرية المنتهية
١٥ــ الكسور العشرية الدورية
١٦ــ الخطأ المئوي
١٧ــ النسبة والتناسب
١٨ــ الوسط المتناسب بين عددين
١٩ــ ربح المساهم (ربح حامل الاسهم)
٢٠ــ البديهية
٢١ــ الموضوعة
٢٢ــ زاوية حادة
٢٣ــ زاوية منفرجة
٢٤ــ زاوية منعكسة
٢٥ــ قطعة دائرة
٢٦ــ قطاع دائرة
٢٧ــ المعاليم
٢٨ــ المجاهيل
٢٩ــ زاويتان متتامتان
٣٠ــ زاويتان متكاملتان
٣١ــ مضلع متساوى الاضلاع
٣٢ــ مثلث متساوى الساقين
٣٣ــ المعين
٣٤ــ المحل الهندسي
٣٥ــ المستقيم القاطع للدائرة
٣٦ــ ازالة وادخال الاقواس
٣٧ــ نقل حدود المعادلة من جهة الى الجهة الاخرى
٣٨ــ متطابقة
٣٩ــ متباينة

ــ يتبع ــ

الاسم:
الرقم:

(25 marks)

(II) Fill in the blanks in the following equations:-

1. one furlong = ( ) chains = ( ) mile
2. one chain = ( ) yards = ( ) links
3. one statute mile = ( ) yds. = ( ) ft.
4. one nautical mile = ( ) ft.
5. one sq. chain = ( ) sq. yds.
6. one acre = ( ) sq. ch. = ( ) sq. yds.
7. one gallon = ( ) pints
8. one bushel = ( ) gallons = ( ) pecks
9. one English ton = ( ) lbs. = ( ) kilograms
10. one English ton = ( ) cwt. = ( ) qr. = ( ) stones.

(25 marks).

الرقم:
الاسم:

٤٠ــ مقدار جبرى متجانس

٤١ــ درجة المقدار الجبرى

٤٢ــ المعامل الحرفي

٤٣ــ مقدار جبرى من الدرجة الثانيةــ

٤٤ــ ان حدى الكسر هما بسطه ومقامه

٤٥ــ في كل عملية قسمة يوجد مقسوم ومقسوم عليه وناتج قسمة وفي بعض الحالات باق للقسمة .

٤٦ــ ان الاعمدة المنصفة لاضلاع مثلث تلتقي في مركز الدائرة المرسومة .....

٤٧ــ ان الخطوط المتوسطة في المثلث تلتقي في نقطة واحدة تقسم كلا منها الى ثلثين من جهة الراس وثلث من جهة القاعدة. وتسمى هذه النقطة مركز ثقل المثلث.

٤٨ــ نقيس طول مستقيم فنجد انه يساوى ٥ر٦١سم. ثم نجد فيما بعد ان طوله المضبوط ٦٠سم.
وفي هذه الحالة نقول ان الخطأ المطلق هو .....
والخطأ النسبي هو ..... والخطأ المئوى هو ...

٤٩ــ ان قيمة المقدار ٧٢ر٥٣٠٩ لاقرب اربعة ارقام معنوية هي .....

٥٠ــ ان المعادلة ٣س ــ ٢س ص + ص = ٥ع ــ سع هي معادلة من الدرجة ..... في .....
..... مجاهيل.

(75 marks)

(II) Fill in the blanks in the following equations:-

1. one furling = ( ) chains= ( ) mile
2. one chain = ( ) yards = ( ) links
3. one statute mile = ( ) yds. = ( )
4. one nautical mile = ( ) ft.
5. one sq. chain = ( ) sq. yds.
6. one acre =( ) sq. ch. = ( ) sq. yds.
7. one gallon = ( ) pints
8. one bushel = ( ) gallons = ( ) pecks
9. one English ton = ( ) lbs. = ( ) kilograms
10. one English ton = ( ) cwt. = ( ) qr. = ( ) stones.

الرقم:
الاسم:

SHAMASH SECONDARY SCHOOL

Monthly Examination, November 1968

Subject: General Mathematics — Date: 18/11/1968.

Class : 4th Year Secondary — Time: 8:30 - 10:00 a.m.

--- --- --- ---

1. Give the English Equivalent of the followinh, filling the blanks in this sheet and hand it over with your examination book.

١ــ ارقام
٢ــ مراتب
٣ــ الطرح
٤ــ العوامل
٥ــ اس القوة
٦ــ مضاعف
٧ــ اعداد زوجية متتالية
٨ــ اعداد فردية متتالية
٩ــ الجزء الصحيح من العدد
١٠ــ اعداد اولية
١١ــ المقام المشترك الاصغر
١٢ــ كسر لفظي
١٣ــ مقلوب العدد
١٤ــ الكسور العشرية المنتهية
١٥ــ الكسور العشرية الدورية
١٦ــ الخطأ المئوى
١٧ــ النسبة والتناسب
١٨ــ الوسط المتناسب بين عددين
١٩ــ ربح المساهم (ربح حامل الاسهم)
٢٠ــ البديهية
٢١ــ الموضوعة
٢٢ــ زاوية حادة
٢٣ــ زاوية منفرجة
٢٤ــ زاوية منعكسة
٢٥ــ قطعة دائرة
٢٦ــ قطاع دائرة
٢٧ــ المعاليم
٢٨ــ المجاهيل
٢٩ــ زاويتان متتامتان
٣٠ــ زاويتان متكاملتان
٣١ــ مضلع متساوى الاضلاع
٣٢ــ مثلث متساوى الساقين
٣٣ــ الصمين
٣٤ــ الحل الهندسي
٣٥ــ المستقيم القاطع للدائرة
٣٦ــ ازالة وادخال الاقواس
٣٧ــ نقل حدود المعادلة من جهة الى الجهة الاخرى
٣٨ــ متطابقة
٣٩ــ متباينة

ــ يتبع ــ

(II) Fill in the blanks in the following equations:-

1. one furlong = ( ) chains = ( ) mile
2. one chain = ( ) yards = ( ) links
3. one statute mile = ( ) ft.
4. one nautical mile = ( ) ft.
5. one sq. chain = ( ) sq. yds.
6. one acre = ( ) sq. ch. = ( ) sq. yds.
7. one gallon = ( ) pints
8. one bushel = ( ) gallons = ( ) pecks
9. one English ton = ( ) lbs. = ( ) kilograms
10. one English ton = ( ) cwt. = ( ) qr. = ( ) stones.

(25 marks)

الرقم:
الاسم:

٤٠ــ مقدار جبرى متجانس

٤١ــ درجة المقدار الجبرى

٤٢ــ المعامل الحرفي

٤٣ــ مقدار جبرى من الدرجة الثانية ــ

٤٤ــ ان حدى الكسر هما بسطه ومقامه

٤٥ــ في كل عملية قسمة يوجد مقسوم ومقسوم عليه وناتج قسمة وفي بعض الحالات باق للقسمة .

٤٦ــ ان الاعمدة المنصفة لاضلاع مثلث تلتقي في مركز الدائرة المرسومة .....

٤٧ــ ان الخطوط المتوسطة في المثلث تلتقي في نقطة واحدة تقسم كلا منها الى ثلثين من جهة الر
وثلث من جهة القاعدة. وتسمى هذه النقطة مركز ثقل المثلث.

٤٨ــ نقيس طول مستقيم فنجد انه يساوى ٥ر٦١سم. ثم نجد فيما بعد ان طوله المضبوط ٦٠سم.
وفي هذه الحالة نقول ان الخطأ المطلق هو .....
والخطأ النسبي هو ..... والخطأ المئوى هو ...

٤٩ــ ان قيمة المقدار ٧٢ر٥٣٠٩ لاقرب اربعة ارقام معنوية هي .....

٥٠ــ ان المعادلة ٣س ــ ٢س + ص = ٥ع ــ سع هي معادلة من الدرجة ..... في .....
..... مجاهيل.

(75 marks)

(II) Fill in the blanks in the following equations:-

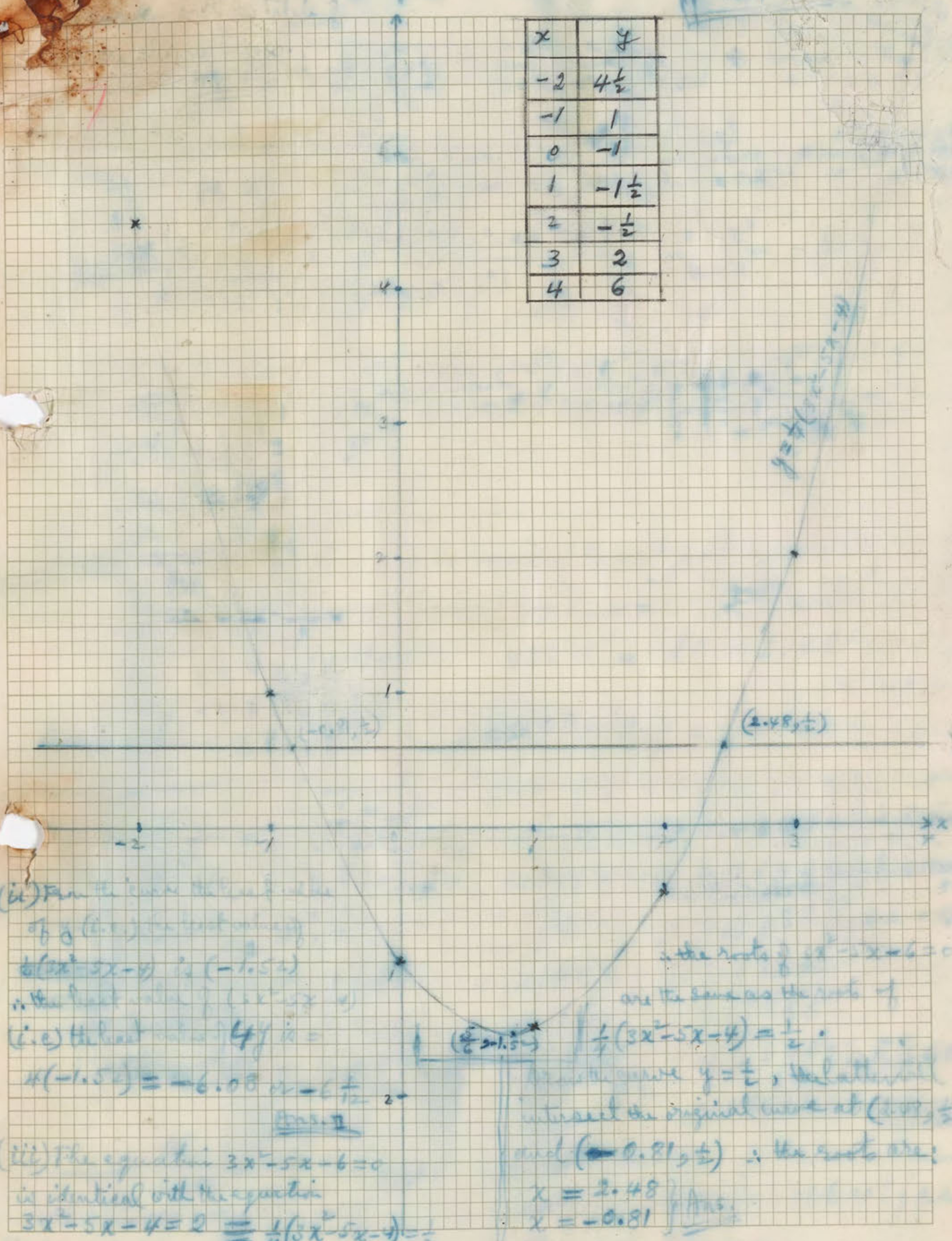
1. one furlong = ( ) chains= ( ) mile
2. one chain = ( ) yards = ( ) links
3. one statute mile = ( ) yds. = ( )ft.
4. one nautical mile = ( ) ft.
5. one sq. chain = ( ) sq. yds.
6. one acre =( ) sq. ch. = ( ) sq. yds.
7. one gallon = ( ) pints
8. one bushel = ( ) gallons = ( ) pecks
9. one English ton = ( ) lbs. =( ) kilograms
10. one English ton = ( ) cwt. = ( ) qr. = ( ) stones.

(25 marks).

$y = \frac{1}{4}(3x^2-5x-4)$

Final Exam in Algebra

x	y
-2	$4\frac{1}{2}$
-1	1
0	-1
1	$-1\frac{1}{2}$
2	$-\frac{1}{2}$
3	2
4	6



(ii) ...

$4(-1.52) = -6.08$ or $-6\frac{1}{12}$

Ans. II

(iii) The equation $3x^2-5x-6=0$ is identical with the equation $3x^2-5x-4=2 \equiv \frac{1}{4}(3x^2-5x-4)=\frac{1}{2}$

∴ the roots of $3x^2-5x-6=0$ are the same as the roots of $\frac{1}{4}(3x^2-5x-4)=\frac{1}{2}$.

... $y=\frac{1}{2}$, ... intersect the original curve at $(2.48, \frac{1}{2})$ and $(-0.81, \frac{1}{2})$ ∴ the roots are:

$x = 2.48$
$x = -0.81$ } Ans.

[illegible] Final Exam

14/5/1968

Let the distance ridden be = x miles, also $\frac{1}{2}$ minute $= \frac{1}{120}$ hours.

$$\frac{x}{3\times10} + \frac{x}{3\times9} + \frac{x}{3\times8} = \frac{x}{2\times10} + \frac{x}{2\times8} - \frac{1}{120}$$

$$\therefore \frac{x}{30} + \frac{x}{27} + \frac{x}{24} = \frac{x}{20} + \frac{x}{16} - \frac{1}{120}$$ the least common denominator is: $2^4 \times 3^3 \times 5$,

$$\text{or } \frac{x}{2\times3\times5} + \frac{x}{3^3} + \frac{x}{2^3\times3} = \frac{x}{2^2\times5} + \frac{x}{2^4} - \frac{1}{2^3\times3\times5}$$ multiplying all the equation by $2^4 \times 3^3 \times 5$ we obtain:

$$\therefore 72x + 80x + 90x = 108x + 135x - 18 \quad \text{or} \quad x = 18 \text{ Ans.}$$

4(i) the sum $S_n = \frac{1}{3}n(4n^2-1)$ $\therefore$ when $n=1$ $S_1 = \frac{1}{3}(4-1) = \frac{1}{3}(3) = 1 = 1^{st}$ term

when $n = 2$, $S_2 = \frac{1}{3}\cdot2(4\times4-1) = \frac{2}{3}\times15 = 10$ = sum of $1^{st} + 2^{nd}$ terms.

$\therefore 1^{st}$ term $= 1$
2nd " $= 10 - 1 = 9$ } Ans.

(ii) 2s. 3d. = 27d. $\therefore$ the boring of the 1^{st} foot costs 27d.
" " 2nd " " 28d. + so on

$\therefore$ we have an A.P. in which $a = 27$, $d = 1$, $n = 400$

$\therefore l = a + (n-1)d = 27 + 399 = 426$ d. = £1. 15s. 6d. cost of boring the last foot

$$S = \frac{n}{2}(a+l) = \frac{400}{2}(27+426) = 200\times453 = 90600 \text{ d. (} £1 = 240\text{d.)}$$

$$\therefore S = \frac{90600}{240} = £377\tfrac{1}{2} = £377.\ 10s.$$ cost of boring the entire well Ans.

(iii) $l_3 = 18$, $l_5 = 40.5$; $a = ?$, $S_6 = ?$

$$\frac{40.5 = ar^4}{18 = ar^2} \quad \therefore r^2 = \frac{81}{2}\cdot\frac{1}{18} = \frac{81}{36}$$

$$\therefore r = \pm\frac{9}{6} = \pm\frac{3}{2}$$

Since all the terms of the G. P. are positive $\therefore r = \frac{3}{2}$

$\therefore 18 = a(\frac{3}{2})^2$ $\therefore a = \frac{4\times18}{9} = 8$ Ans. 1

$$S_6 = \frac{a(r^6-1)}{r-1} = \frac{8[(\frac{3}{2})^6-1]}{\frac{3}{2}-1} = \frac{8[\frac{729}{64}-1]}{\frac{1}{2}} = 16\left(\frac{729-64}{64}\right)$$

$$\therefore S_6 = \frac{16\times665}{64} = \frac{665}{4} = 166\tfrac{1}{4} \quad \text{Ans. 2}$$

5.(i) Draw the graph of $y = \frac{1}{4}(3x^2-5x-4)$ from $x = -2$ to $x = 2$ using 1 inch = 1 unit on both axes

(ii) Find the least value of $3x^2-5x-4$

(iii) Solve the equation $3x^2-5x-6=0$

x	y
-2	[illegible]
-1	1
0	-1
1	[illegible]
2	[illegible]
3	[illegible]
4	6

Final Examination in Algebra, [illegible] year

14/5/1968

$[illegible] - 8y^9 = x^3(x^6+y^6) - 8y^3(x^6+y^6)$

$= (x^6+y^6)(x^3-8y^3) = (x^2+y^2)(x^4-x^2y^2+y^4)(x-2y)(x^2+2xy+4y^2)$ Ans.

$[illegible] \div \frac{a(a+x)}{a-x} \left\{ \frac{x^2(a^2-x^2)}{(a+x)(a^2-ax+x^2)} \div \frac{a^2(a^2-2ax+x^2)}{a^2-ax+x^2} \right\}$

$= \left\{ \frac{(a-x)(a+x)(a^2+x^2)}{(a-x)^2} \times \frac{(a-x)}{a(a+x)} [illegible] \frac{(a-x)(a+x)}{(a+x)(a^2-ax+x^2)} \times \frac{a^2-ax+x^2}{a^2(a-x)^2} \right\}$

$= \frac{a^3(a+x)^2(a-x)^3(a^2+x^2)(a^2-ax [illegible])}{a^3(a+x)^2(a-x)^4(a^2-ax+x^2)} = \frac{[illegible] x^2}{x}$ Ans.

$x^{\frac{5}{2}} + 2x^{\frac{3}{2}} - [illegible] - x^{\frac{?}{2}} + 3x^2 + x^{\frac{3}{2}} + x + 3x^{\frac{1}{2}} + B \;\Big|\; x + x^{\frac{1}{2}} - 2$

$ 2x^3 - x^2 + x + 3$

$2x^{\frac{5}{2}} + 2x^2 - 4x^{\frac{3}{2}}$

$-x^3 - x^{\frac{5}{2}} + 2x^2$

$-x^3 - x^{\frac{5}{2}} + 2x^2$

$x^2 + x^{\frac{3}{2}} + x$

$x^2 + x^{\frac{3}{2}} - 2x$

$3x + 3x^{\frac{1}{2}} + B$

$3x + 3x^{\frac{1}{2}} - 6$

Remainder $= B + 6 = 0 \quad \therefore B = -6$ Ans.

$x = \sqrt[7]{\frac{(0.1023)^3 (\cos^2 41^\circ 28')}{(1.007)^2 (\tan^5 47^\circ 51')}}$

$\log 0.1023 = \bar{1}.0098$	$3 \log 0.1023 = \bar{3}.0294$	$2 \log 1.007 = 0.0058$
$\log \cos 41^\circ 28' = \bar{1}.8747$	$2 \log \cos 41^\circ 28' = \bar{1}.7494$	$5 \log \tan 47^\circ 51' = 0.2165$
$\log 1.007 = 0.0029$	$\log$ Num. $= \bar{4}.7788$	$\log$ Den. $= 0.2223$
$\log \tan 47^\circ 51' = 0.0433$	$\log$ Den. $= 0.2223$	
	$7 \log x = \bar{4}.5565$	
	$\log x = \bar{1}.50807 = \bar{1}.5081$	
	$x = 0.3222$ Ans.	

(ii) $(31.01)^{3x-1} = 104(2.003)^{2x+1} \quad \therefore (3x-1)\log 31.01 = \log 104 + (2x+1)\log 2.003$

$3x \log 31.01 - \log 31.01 = \log 104 + 2x \log 2.003 + \log 2.003$

$3x \log 31.01 - 2x \log 2.003 = \log 104 + \log 2.003 + \log 31.01$

$\therefore x(3\log 31.01 - 2\log 2.003) = \log 104 + \log 2.003 + \log 31.01 \quad \therefore x = \frac{\log 104 + \log 2.003 + \log 31.01}{3\log 31.01 - 2\log 2.003}$

$\log 104 = 2.0170$	2.0170
$\log 2.003 = 0.3016$	0.3016
$\log 31.01 = 1.4915$	1.4915
$2 \log 2.003 = 0.6032$	3.8101
$3 \log 31.01 = 4.4745$	

$\therefore x = \frac{2.0170 + 0.3016 + 1.4915}{4.4745 - 0.6032}$

$\therefore x = \frac{3.8101}{3.8713} = 0.98419 = 0.9842$

4.4745	
0.6032	
3.8713	

Shamash Secondary School
Mid-Year Examination, Feb.1968

Subject: Algebra
Class: 4th Year, Scientific Section.

Date: 7/2/1968
Time: 8:30 - 11:00 a.m.

Attempt all questions:

1. (a) Resolve into factors:

(i) $(2a+b)x^2 - (a-b)x - (a + 2b)$ (4 marks)

(ii) $201x^2 - 99x - 102$ (4 ")

(iii) $x^9 - 64x^3 - x^6 + 64$ (4 ")

(b) Show that any common factor of A and B is also a factor of mA & nB (8 marks)

2. (a) The following equation is true for all values of x:

$(2x-3)^2-c = 2Ax^2 - 4Bx$. Find the values of A,B and C. (8 ")

(b) Of the following three equations, one is always true, one is sometimes true and one is never true. Find which is which, giving your reasons:

(i) $3x(x-4)+x = 5(x^2-1)+13-11x$ (4 marks)

(ii) $x^2(2x-5)+3(x-1) = 2x^3-x(5x-3)-3$ (4 ")

(iii) $x(x^2-1)+2(1+x)(1-x) = o$ (4 ")

3. (i) Solve the equation: $3x^3-14x^2+32 = o$ (10 ")

(ii) Solve the two simultaneous equations:

$x+y+2xy+x^2+y^2 = o$ .....(1)

$x-y-2xy+x^2+y^2 = 6$ .....(2) (10 ")

4. (i) Running separately, two taps can fill a bath with water in "a" and "b" minutes respectively. Prove that they take $\frac{ab}{a+b}$ minutes to fill it when running together. (10 marks)

(ii) If, when they are running separately, the first tap can fill the bath in 7 minutes less time than the second, and when they are running together they fill it in 12 minutes, find the values of "a" and "b". (10 marks)

5. (i) In an examination taken by both boys and girls, 41 candidates out of every 68 pass. Five boys out of every 8 pass and 7 girls out of every 12 pass. Find the ratio of boy candidates to girl candidates.

(ii) If 168 girls passed the examination, find the total number of candidates.

---

4) (i) The 1st tap fills the bath in "a" minutes ∴ it fills $\frac{1}{a}$ of the bath in one minute

The 2nd " " " " " "b" " ∴ " " $\frac{1}{b}$ " " " " "

∴ both taps together fill $\left(\frac{1}{a}+\frac{1}{b}\right)$ of the bath in one minute

" " " " " $\left(\frac{a+b}{ab}\right)$ " " " " " "

" " " " " $\left(\frac{1}{ab}\right)$ " " " " " $\left(\frac{1}{a+b}\right)$ minutes

" " " " " $\left(\frac{ab}{ab}\right)$ " " " fill the bath in $\left(\frac{ab}{a+b}\right)$ minutes. Ans.

(ii) When running separately the 1st tap fills the bath in 7 minutes less time than the 2nd tap ∴ $b-a=7$ ..... ①

Also, when running together, they both fill the bath in $\left(\frac{ab}{a+b}\right)$ minutes ∴ $\frac{ab}{a+b}=12$ ..... ②

from ① $b=a+7$ and from ② $ab=12a+12b$

∴ $a(a+7)=12a+12(a+7)$ or $a^2+7a=12a+12a+84$

$a^2-17a-84=0$ ∴ $(a+4)(a-21)=0$ ∴ $a=-4$ to be discarded

∴ $b=a+7=21+7=28$ minutes ∴ $\left\{\begin{array}{l} a=21 \text{ minutes} \\ \text{and } b=28 \text{ minutes}\end{array}\right.$ Ans.

---

5 (i) Let the number of boy candidates in any one case be $= b$ and [illegible] girl [illegible] in the same case be $= g$

∴ $\frac{5b}{8}+\frac{7g}{12}=\frac{41}{68}(b+g)$ ∴ $5\times 17\times 3b+7\times 2\times 17g=6\times 41(b+g)$

$255b+238g=246b+246g$ or $9b=8g$ ∴ $\frac{b}{g}=\frac{8}{9}$

∴ $\frac{\text{No. of boy candidates}}{\text{No. of girl candidates}}=8:9$ Ans.

(ii) When the No. of girl candidates who passed the examination is equal to 168; then in the above relation we get: $\frac{7}{12}g=168$

∴ $g=\frac{168\times 12}{7}=24\times 12=288$

$b=\frac{8}{9}g=\frac{8}{9}\times 288=8\times 32=256$ } ∴ $b+g=256+288=544$

∴ Total number of candidates = 544. Answer

(i) $3x(x-1)+x = 5(x^2-1)+13-11x$ [illegible]

$\therefore 3x^2-3x+x = 5x^2-5+13-11x$ or $2x^2 = -8$ or $x^2 = -4$ [illegible]

[illegible]

(ii) $x^2(2x-5)+2\left(\frac{x-1}{2}\right) = 2x^3 - x(5x-1) - 1$

$\therefore 2x^3-5x^2+x-1 = 2x^3-5x^2+x-1$ [illegible] an identity and [illegible] therefore true for all values of x

(iii) $x(x^2-1)+2(1+x)(1-x)=0$

$\therefore x(x+1)(x-1)+2(x+1)(1-x)=0$ [illegible]

$\therefore (x+1)[x(x-1)+2(1-x)]=0$ or $(x+1)[x(x-1)-2(x-1)]=0$

$\therefore (x+1)(x-1)[x-2]=0$ $\therefore$ $\left.\begin{array}{l}x=1\\x=-1\\x=2\end{array}\right\}$ it is a conditional equation which is true for these values only. In other words it is an equation not [illegible].

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2. (i) $3x^3-14x^2+32=0$ By trial [illegible] $x=2$ satisfies the equation since $3(2)^3-14(2)^2+32 = 24-56+32=0$ $\therefore x=2$ is a root and hence $(x-2)$ is a factor

$\therefore 3x^3-14x^2+32 = (x-2)(3x^2-8x-16)=0$

$\therefore (x-2)(3x+4)(x-4)=0$ $\therefore$ $\left.\begin{array}{l}x=2\\x=4\\x=-\frac{4}{3}\end{array}\right\}$ Ans.

(ii) $x+y+2xy+x^2+y^2=0$ ...... ① $\therefore (x+y)+(x+y)^2=0$ (1a)

$x-y+2xy+x^2+y^2=6$ ...... ② $\therefore (x-y)+(x-y)^2=6$ (2a)

from (1a) $(x+y)[1+x+y]=0$ $\therefore x+y=0$ or $y=-x$ ...... ③

also $1+x+y=0$ $\therefore y=-x-1$ ...... ④

substitute from ③ into (2a), then $(x+x)+(x+x)^2=6$ or $4x^2+2x-6=0$

$\therefore 2x^2+x-3=0$ or $(2x+3)(x-1)=0$ $\therefore$ $\begin{array}{l}x=1\\x=-\frac{3}{2}\end{array}$ from ③ $\therefore$ $\begin{array}{l}y=-1\\y=\frac{3}{2}\end{array}$

$\therefore \left.\begin{array}{l}x=1\\y=-1\end{array}\right\}$ Ans. 1 $\left.\begin{array}{l}x=-\frac{3}{2}\\y=\frac{3}{2}\end{array}\right\}$ Ans. 2

substituting from ④ into (2a) we get $(x+x+1)+(x+x+1)^2=6$ $\therefore 2x+(2x+1)^2=5$

$\therefore 4x^2+6x-4=0$ or $2x^2+3x-2=0$ $\therefore (2x-1)(x+2)=0$ $\therefore x=\frac{1}{2}$ and $y=-\frac{3}{2}$, $x=-2$ and $y=1$

$\left.\begin{array}{l}x=\frac{1}{2}\\y=-\frac{3}{2}\end{array}\right\}$ Ans. 3 $\left.\begin{array}{l}x=-2\\y=1\end{array}\right\}$ Ans. 4

(iii) $x^9 - 64x^3 - x^6 + 64 = x^9 - x^6 - 64(x^3 - 1) = x^6(x^3-1) - 64(x^3-1)$

$= (x^3-1)(x^6-64) = (x-1)(x^2+x+1)(x^3-8)(x^3+8)$

(b) Let ... and let $A = aF$ and $B = bF$.

... F is also a common factor ...

2.(a) ... $A + B - C = 5$ ...... ①

... $A - B - C = 3$ ...... ②

...... ③

Shamash Secondary School
Final Examination, May 1968

Subject: Mathematics
Class: 4th Scientific Year

Date: 14/5/1968
Time: 8:00-11:00 a.m.

All questions are to be attempted.

1. (i) Resolve into four factors: $x^9+x^3y^6-8x^6y^3-8y^9$

(6 marks)

(ii) Simplify: $\left\{\frac{a^4-x^4}{a^2-2ax+x^2} \div \frac{a^2+ax}{a-x}\right\} \times \left\{\frac{a^5-a^3x^2}{a^3+x^3} \div \frac{a^4-2a^3x+a^2x^2}{a^2-ax+x^2}\right\}$

(7 marks)

(iii) Find the value of B if $2x^4+2x^{\frac{7}{2}}-5x^3-x^{\frac{5}{2}}+3x^2+x^{\frac{3}{2}}+x^{\frac{1}{2}}+3+B$ is exactly divisible by $x+x^{\frac{1}{2}}-2$

(7 marks)

2. (i) Compute by logarithms the value of x, if $x=\sqrt[7]{\frac{(0.1023)^3.\cos^2 41°28'}{(1.007)^2(\tan^5 47°51')}}$

arranging your work neatly.

(10 marks)

(ii) Solve for x the equation: $(31.01)^{3x-1} = 104(2.003)^{2x+1}$

(10 marks)

3. I rode one third of a journey at 10 miles an hour, one third more at 9, and the rest at 8 miles an hour. If I had ridden half the journey at 10, and the other half at 8 miles per hour, I should have been half a minute longer on the way. What distance did I ride ?

(20 marks)

4. (i) The sum of n terms of a series is $\frac{1}{3}n(4n^2-1)$. Find the first two terms.

(6 marks)

(ii) In boring a well 400 ft deep the cost is 2s. 3d. for the first foot and an additional penny for each subsequent foot. What is the cost of boring the last foot, and also of boring the entire well ?

(7 marks)

(iii) The third term of a geometric series, in which all the terms are positive, is 18 and the fifth term is 40.5. Find the first term and the sum of the first six terms.

(7 marks)

5. (i) Draw the graph of $y=\frac{1}{4}(3x^2-5x-4)$ for values of x from -2 to +3, using a scale of 1 inch to 1 unit on each axis.

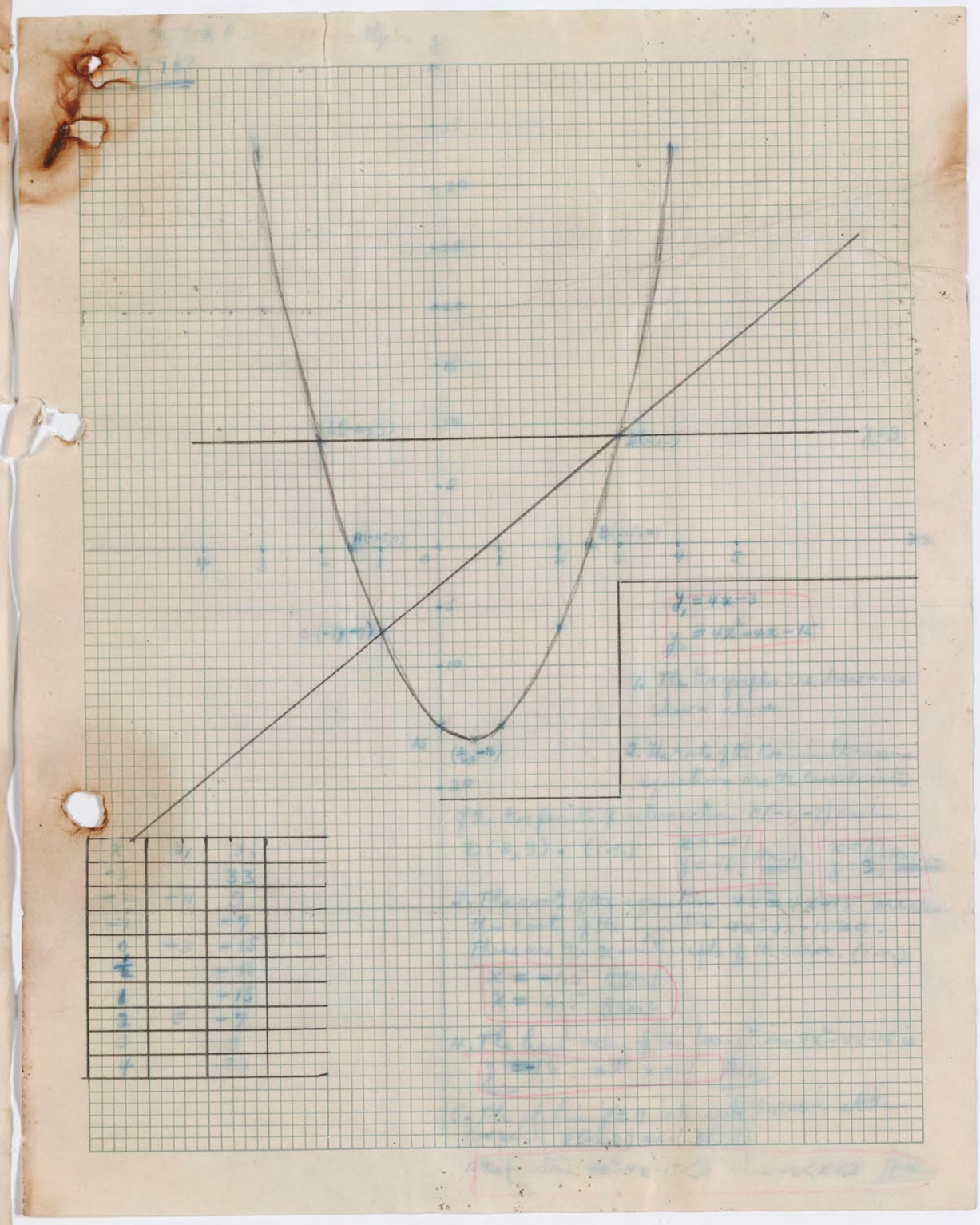
(7 marks)

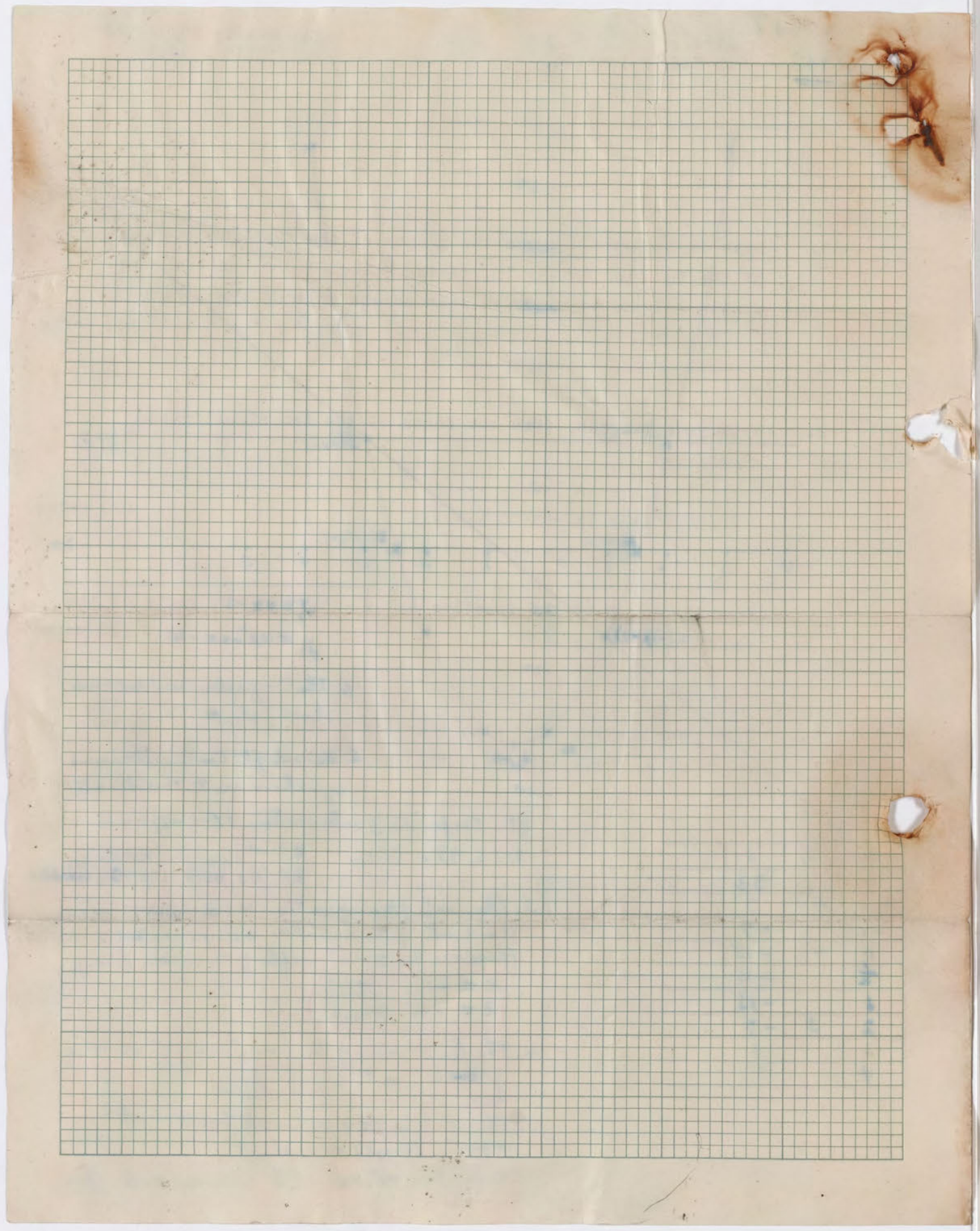
(ii) Use your graph to find the least value of $3x^2-5x-4$.

(6 marks)

(iii) By drawing the appropriate straight line on your graph solve the equation $3x^2-5x-6=0$.

(7 marks)





Shamash Secondary School
3rd Quarter Examination, March, 1968

Subject: Algebra
Class: 4th Year Scientific

Date: Sunday 24/3/1968
Time: 10:15 - 11:45

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1. Draw on the same diagram the graphs of the function $4x-3$ and of the function $4x^2-4x-15$, taking ½ inch as one unit on the x-axis and one tenth of an inch as one unit on the y-axis.

(20 marks)

2. From your diagram, find the roots of the two simultaneous equations:

$y = 4x-3$ .....(1)
$y = 4x^2-4x-15$ .....(2)

(20 marks)

3. From the graph of the function $4x^2-4x-15$, find the roots of the equation $4x^2 = 4x + 15$

(20 marks)

4. From your diagram find also the least value of $4x^2-4x-15$, and the value of x corresponding to the least value of the function.

(20 marks)

5. By drawing an additional graph, find the values of x for which the expression $4x^2-4x-15$ is always less than 9.

(20 marks).

SHAMASH SECONDARY SCHOOL

End Quarter Examination

Subject: Algebra — Date : 31/12/1967.
Class : 4th Year, Scientific Section. — Time : 10:15-11:45 a.m.

Attempt all questions:

1. (i) Resove into five factors : $X^{12} - Y^{12}$ (10 marks)

(ii) Find the value of 'A' which will make the expression $6x^3 + Ax^2 + x - 6$ divisible by $(x + 3)$ and find the other factors. (10 marks)

2. Find the square root of :

$$\frac{9}{4}x^6 - 2x^5 + \frac{74}{45}x^4 - \frac{61}{30}x^3 + \frac{62}{75}x^2 - \frac{2}{5}x + \frac{1}{4}.$$ (20 marks)

3. (i). Reduce to simplest form : $\dfrac{1 + \frac{a-b}{a+b}}{1 - \frac{a-b}{a+b}} + \dfrac{1 + \frac{a^2-b^2}{a^2+b^2}}{1 - \frac{a^2-b^2}{a^2+b^2}}$ (10 marks)

(ii). Solve the equation : $\frac{5x-8}{x-2} + \frac{6x-44}{x-7} - \frac{10x-8}{x-1} = \frac{x-8}{x-6}$. (10 marks)

4. Find the values of x, y, and z from the following equations :

$$3x - 2y + 4z = 3y - 2x + 7 = 7x + 2z - 2 = 11 .$$ (20 marks)

5. A basket of eggs is emptied by one person taking half of them and one more, a second person taking half of the remaiander and one more, and a third person taking half of the remainder and six more. How many did the basket contain at first? (20 marks)

$$= \frac{3a}{2b} \div \frac{3a^2}{2b} = \frac{1}{1} \times \frac{b^2}{a} = \frac{b}{a}$$

(ii) $\frac{5x-5}{x-2} + \frac{(x-1)^2}{x-1} - \frac{16x-8}{x^2-3} = \frac{2x+2}{x-2}$

$\frac{5(x-1)}{x-2} + \frac{(x-1)^2}{x-1} - \frac{16(x-1)+8}{x^2-3} = \frac{x-6-2}{x-2}$

$5 + \frac{3}{x-2} + 1 - \frac{3}{x-1} - 16 - \frac{8}{x^2-3} = 1 - \frac{3}{x-2}$

$\frac{3}{x-2} - \frac{3}{x-1} = \frac{3}{x-1} - \frac{3}{x-2}$ $\quad \frac{1}{x-1} - \frac{1}{x-1} = \frac{1}{x-1} - \frac{1}{x-2}$

$\frac{x-1-(x-2)}{(x-2)(x-1)} = \frac{x-2-(x-1)}{(x-1)(x-2)}$ $\quad \frac{-1}{(x-1)(x-2)} = \frac{-1}{(x-1)(x-2)}$

$(x-2)(x-1) = (x-1)(x-2)$

$x^2 - 3x + 2 = x^2 - 7x + 6 \quad \therefore 4x = 4 \quad \therefore x = 1$ Ans.

(10 marks)

[illegible]

5\. [illegible]

∴ 1st person takes [illegible] Remainder [illegible]

∴ 2nd " takes [illegible] +1 [illegible] Remainder = [illegible]

[illegible]

[illegible] person takes [illegible]

[illegible]

$\therefore x = \frac{x+2}{2} + \frac{x+4}{4} + \frac{x+48}{8}$ [illegible]

$\therefore x = 54$ [illegible]

(20 marks)

[illegible]

Solution to Monthly Quiz — 4th year Secondary, 1967 — 12/11/1967

II. (a) $k = \frac{20ab}{4a+5b}$

(i) $20ab = 4ak + 5bk \therefore 20ab - 4ak = 5bk$

$\therefore 4a(5b-k) = 5bk \therefore a = \frac{5bk}{4(5b-k)}$ Ans. 1 — 5

(ii) $20ab - 5bk = 4ak \therefore 5b(4a-k) = 4ak$

$\therefore b = \frac{4ak}{5(4a-k)}$ Ans. 2 — 5

$$\sqrt{\frac{k-4a}{k-5b}} = \sqrt{\frac{\frac{20ab}{4a+5b} - 4a}{\frac{20ab}{4a+5b} - 5b}} = \sqrt{\frac{20ab - 16a^2 - 20ab}{20ab - 20ab - 25b^2}}$$

$$= \sqrt{\frac{16a^2}{25b^2}} = \frac{4a}{5b}$$ Ans. 3 — 5

(b) $a = 0$, $b = 1$, $c = -2$, $d = 2$ — 10

$$(3abc - abcd)\sqrt[3]{a^2bc - c^3bd + 3} = [(0) - 2(1)(-4)(3)]\sqrt[3]{0 - (-2)^3(1)(3) + 3}$$

$$= 12\sqrt[3]{24+3} = 12\sqrt[3]{27}$$ Ans. $= 36$ Ans.

(c) $\frac{1}{2}x^2 - ax - \frac{2}{3}a^2$ — 15

$\times \quad \frac{3}{4}x^2 - \frac{1}{2}ax + \frac{1}{3}a^2$

$\frac{3}{8}x^4 - \frac{3}{4}ax^3 - \frac{1}{2}a^2x^2$

$\qquad - \frac{3}{4}ax^3 + \frac{1}{2}a^2x^2 + \frac{1}{3}a^3x$

$\qquad\qquad \frac{1}{6}a^2x^2 - \frac{1}{3}a^3x - \frac{2}{9}a^4$

$\frac{9}{8}x^4 - \frac{3}{2}ax^3 + \frac{1}{2}a^2x^2 - \frac{2}{9}a^4$

40

الرقـــم :

Shamash Secondary School
Monthly Quiz.

الاســـم :

Subject: Algebra — Date: 12/11/1967
Class: 4th Year Secondary — Time: 1:15- 11:45 a.m.

I. Give the English equivalent to the following and fill in the blanks in this sheet, handing it back with your answer book:

١ــ في المقدار اس يسمى ا معامل حرفي

1.

٢ــ ننقل الحدود من طرف الى الطرف الثاني للمعادلة ونجمع الحدود المتشابهة

2.

٣ــ نتخلص من الكسور

3.

٤ــ نوحد مقامات الكسور بابسط مقام مشترك

4.

٥ــ ان قيمة المقدار ٤٥٠٣٫٨٤ لاقرب اربعة ارقام معنوية هي

5.

٦ــ ان حدّى الكسر هما بسطه ومقامه

6.

٧ــ نقيس طول مستقيم فنجد انه يساوى ٦١٫٥ سم بينما طوله المضبوط هو ٦٠ سم. وفي هذه الحالة نقول ان الخطأ المطلق هو والخطأ النسبي هو والخطأ المئوى هو

7.

٨ــ ان المعادلة : ٣س٢ ــ ٥ص٢ + ع٢ = ٧ هي معادلة من الدرجة الثانية ذات ثلاثة مجاهيل .

8.

٩ــ في كل عملية قسمة يوجد مقسوم ومقسوم عليه وناتج قسمة ، وفي بعض الحالات باق للقسمة

9.

١٠ــ مقلوب العدد ــ زاويتان متتامتان ــ زاويتان متكاملتان ــ محيط المضلع

10.

(تتبع ص ٢)

الاســـم: الرتبـــة:

4th Year - Algebra -p.2- Monthly Quiz.

II. (a) If $k = \frac{20ab}{4a+5b}$ find (i) "a" in terms of "b" and "k"

(ii) "b" in terms of "a" and "k".

Find also the value of $\sqrt{\frac{k - 4a}{k - 5b}}$ in terms of "a" and "b".

(b) If a = 0, b=1, c=-2, d=3, find the value of

$$(3abc - 2bcd)\sqrt[3]{a^3bc - c^3bd+3}$$

(c) Find the product of $\frac{3}{2}x^2 - ax - \frac{2}{3}a^2$ and $\frac{3}{4}x^2 - \frac{1}{2}ax + \frac{1}{3}a^2$

Shamash Secondary School
Conditional Examination, Sept.1967

Subject: Algebra — Date: 8/9/1967
Class: 4th Year Secondary — Time: 8:00 - 11:00 a.m.

Attempt all questions:

1. (i) Find the value of k if the expression $6x^3-13x^2+18x+k$ is exactly divisible by $2x^2-3x+4$ (10 marks)

(ii) If the n th term of a series is $\frac{2n+1}{2n+3}$ write down the first three terms and express the difference between the n th and (n+1)th terms as a single fraction in its simplest form. (10 marks)

2. (i) If $3x^2-4x+5 = a(x-b)^2+c$ for all values of x, find the values of a, b, and c. (10 marks)

Hence, or otherwise, find the least value of $3x^2-4x+5$.

(ii) Find the lapse of time in minutes between the two instants when the two hands of a watch are at right angles for the 1st and the 2nd time between four O'clock and five O'clock. (10 marks)

3. (i) Compute by logarithms the following expression :

$$\sqrt[9]{\frac{\sin^2 15^\circ 04' \times \cos^3 31^\circ 31'}{(510.7)^2 \times (4.007)^3}}$$ (10 marks)

(ii) Find the value of x from the following equation correct to four significant figures:

$$32^{2x-1} = 64^x \times 40$$ (10 marks)

4. (i) Three times the third term of an arithmetic progression is twice the sixth term. The sum of the first, third and fifth terms is 9. Find:

(a) the ratio of the ninth term to the sixth term,
(b) the sum of the first thirteen terms of the progression. (10 marks)

(ii) The third term of a geometric progression, in which all the terms are positive, is $\frac{2}{3}$ and the sum of the first two terms is 2½.

Find the first term, the common ratio and the fourth term of the progression. (10 marks)

5. (i) Taking 1 inch = 1 unit on the x-axis and 1 inch = 2 units on the y-axis draw the graphs of $y = 4-x^2$ and $4y = 5x + 4$ for values of x from -3 to +3. (8 marks)

(ii) From your graph, find:
a- the range of values of x for which $4-x^2$ is greater than $\frac{5}{4}x+1$, (4 marks)
b- the values of x for which $4-x^2=2.5$, (4 marks)
c- the square root of 5.6. (4 marks)

Conditional examination, September 1967

Subject: Algebra
Class : 4th Year Secondary

Date: 8/9/1967
Time: 8:00 – 11:00 a.m.

Attempt all questions:

1. (i) Find the value of k if the expression $6x^3-15x^2+12x+k$ is exactly divisible by $2x^2-3x+4$ (6 marks)

(ii) If the n^{th} term of a series is $\frac{2n+1}{2n+2}$ write down the first three terms and express the difference between the n^{th} and $(n+1)^{th}$ term as a single fraction in its simplest form. (10 marks)

(i) If $3x^2-4x+c = a(x-b)^2+c$ for all values of x, find the values of 'a', 'b', and 'c'.
Hence, or otherwise, find the least value of $3x^2-4x+5$. (10 marks)

(ii) Find the [illegible] of time in minutes between the two instants when the [illegible] of a watch are at right angles for the 1st and 4th time between [illegible] o'clock and [illegible] four o'clock and five o'clock. (6 marks)

3.(i) Compute [illegible] the following expression:

$$\sqrt{\frac{[illegible]}{(-10.7)^2 \times (4.02)^2}}$$ (10 marks)

(ii) Find the [illegible] of the following equation correct to [illegible] four significant figures

$$[illegible] \cdot 32^{x-1} = 64 \times 40$$ (5 marks)

4. (i) The [illegible] third term of an arithmetic progression is twice the sixth term. The sum of the first, third and fifth terms is 9.
Find:
(a) the ratio of the [illegible] with term to the sixth term,
(b) the sum of the first thirteen terms of the progression. (10 marks)

(ii) The third [illegible] and $\frac{1}{2}$ and 1

5. (i) Taking 1 inch = 1 unit on the x-axis and 1 inch = 2 units on the y-axis draw the graphs of $y = 4 - x^2$ and $4y = 5x + 4$ for values of x from -3 to $+3$. (8 marks)

(ii) From your graph, find:

(a) the range of values of x for which $4 - x^2$ is greater than $\frac{5}{4}x + 1$, (4 marks)

(b) the values of x for which $4 - x^2 = 2.5$, (4 marks)

(c) the square root of 5.6. (4 marks)

5. (i) Taking 1 inch = 1 unit on the x-axis and 1 inch = 2 units on the y-axis draw the graphs of $y = 4 - x^2$ and $4y = 5x + 4$ for values of x from -3 to $+3$. (8 marks)

(ii) From your graph, find:

(a) the range of values of x for which $4 - x^2$ is greater than $\frac{5}{4}x + 1$, (4 marks)

(b) the values of x for which $4 - x^2 = 2.5$, (4 marks)

(c) the square root of 5.6. (4 marks)

Solution to Continuation examination in Algebra, Sept., 1967, 4th year

1. (i)
$$\begin{array}{r|l} 2x^2-3x+4 & 6x^3-13x^2+18x+k \;\big|\; 3x-2 \\ \hline 6x^3-9x^2+12x & \\ \hline -4x^2+6x+k & \\ -4x^2+6x-8 & \\ \hline 8+k & \end{array}$$

$8+k = 0 \quad \therefore k = -8$ Ans.

(ii) n^{th} term $= \frac{2n+1}{2n+3}$ $\therefore$ 1st term $= \frac{2+1}{2+3} = \frac{3}{5}$, 2nd t. $= \frac{2\times2+1}{2\times2+3} = \frac{5}{7}$

3rd term $= \frac{2\times3+1}{2\times3+3} = \frac{7}{9}$

$$(n+1)^{th} \text{ term} - n^{th} \text{ term} = \frac{2(n+1)+1}{2(n+1)+3} - \frac{2n+1}{2n+3} = \frac{2n+3}{2n+5} - \frac{2n+1}{2n+3} =$$

$$= \frac{(2n+3)^2 - (2n+1)(2n+5)}{(2n+5)(2n+3)} = \frac{4n^2+12n+9-(4n^2+12n+5)}{(2n+5)(2n+3)}$$

$$= \frac{4}{(2n+5)(2n+3)} \text{ Ans.}$$

2 (i) $3x^2-4x+5 = a(x-b)+c \quad \therefore 3x^2-4x+5 = ax^2-2abx+ab^2+c$

$\therefore a = 3$, $\therefore -2ab = -4$ or $3b = 2$ or $b = \frac{2}{3}$

$\therefore ab^2+c = 5$ or $3\times\frac{4}{9}+c = 5$ or $c = 5-\frac{4}{3}$ or $c = \frac{11}{3} = 3\frac{2}{3}$

$\therefore a = 3$, $b = \frac{2}{3}$, $c = \frac{11}{3} = 3\frac{2}{3}$ Ans.

Hence $3x^2-4x+5 = 3\left(x-\frac{2}{3}\right)^2 + \frac{11}{3}$ and since $\left(x-\frac{2}{3}\right)^2$ is always positive its least value will be zero when $x = \frac{2}{3}$

$\therefore$ therefore the least value of $3x^2-4x+5$ is $\boxed{\frac{11}{3}}$ when $x = \frac{2}{3}$

(ii) Let the [illegible]

$\therefore x = 10 + \frac{5}{6} + \frac{7}{11}$ or $x = \frac{2}{11}$ [illegible]

$\therefore 12x =$ [illegible]

$\therefore 11x = 360 \quad \therefore x = \frac{360}{11} = 32\frac{8}{11}$ minutes

$\therefore x = 32$ minutes [illegible]

3 (i) $$\sqrt[9]{\frac{\sin^2 15^\circ 04' \times \cos^3 31^\circ 31'}{(510.7)^2 \times (4.007)^3}} = x$$

$\log \sin 15^\circ 04'$	$= \bar{1}.4148$	$2 \log \sin 15^\circ 04'$	$= \bar{2}.8296$
$\log \cos 31^\circ 31'$	$= \bar{1}.9307$	$3 \log \cos 31^\circ 31'$	$= \bar{1}.7921$
$\log 510.7$	$= 2.7082$	$\log$ Num.	$= \bar{3}.6217$
$\log 4.007$	$= 0.6029$	$\log$ Den.	$= 7.2251$
$2 \log 510.7$	$= 5.4164$	$9 \log x$	$= \bar{9}.3966$
$3 \log 4.007$	$= 1.8087$	$\log x$	$= \bar{1}.04407$
$\log$ Den.	$= 7.2251$		$= \bar{1}.0441$
		x	$= 0.1107$ Ans.

(ii) $32^{2x-1} = 64^{x} \cdot 40$ or $2^{5(2x-1)} = 2^{6x} \times 40$ or $2^{5(2x-1)-6x} = 40$

$\therefore 2^{4x+5} = 40$ $\therefore (4x+5)\log 2 = \log 40$

$(4x+5)\log 2 = \log 2^3 + \log 5$ $\therefore (4x+5)\log 2 = 3\log 2 + \log 5$

or $2^{4x+5-2} = 2^2 \times 10$ or $2^{4x+3} = 10$ or $2^{4x+3} = 10$

$(4x+3)\log 2 = \log 10$ $\therefore (4x+3)\log 2 = 1$ $\therefore 4x\log 2 + 3\log 2 = 1$

$\therefore 4x\log 2 = 1 - 3\log 2$ $\therefore x = \frac{1-3\log 2}{4\log 2} = \frac{1-3\times 0.3010}{4\times 0.3010}$

$\therefore x = \frac{1-0.9030}{1.2040} = \frac{0.0970}{1.2040} = \frac{97}{1204} = 0.080564\ldots$

$\therefore x = 0.08056$ Correct to 4 significant figures.

or $2^{4x-5} = 2^2 \times 10$ or $2^{4x-5-2} = 10$ or $2^{4x-7} = 10$

$\therefore (4x-7)\log 2 = 1$ or $4x\log 2 - 7\log 2 = 1$ or $4x\log 2 = 1 + 7\log 2$

or $x = \frac{1+7\log 2}{4\log 2} = \frac{1+7\times 0.3010}{4\times 0.3010} = \frac{1+2.1070}{1.2040} = \frac{3.1070}{1.2040} = \frac{3.107}{1.204}$

$\therefore x = 2.5772\ldots = 2.577$ Correct to 4 significant figures

Ans

4. (i) Let a = 1st term, d = Common difference

$\therefore 3(a+2d) = 2(a+5d)$ or $3a+6d = 2a+10d$ or $\begin{cases} a-4d=0 \ \ldots\ldots (1) \\ a+2d=3 \ \ldots\ldots (2) \end{cases}$

also $a+(a+2d)+(a+4d)=9$ or $3a+6d=9$ or

$\therefore 6d=3 \quad \therefore \boxed{d=\frac{1}{2}} \quad \therefore \boxed{a=2}$

(a) $\therefore \dfrac{9\text{th term}}{6\text{th term}} = \dfrac{a+8d}{a+5d} = \dfrac{2+4}{2+\frac{5}{2}} = \dfrac{6}{4\frac{1}{2}} = \dfrac{12}{9} = \boxed{\dfrac{4}{3}}$ Ans. 1

(b) $S_{13} = \frac{13}{2}\{2a+(n-1)d\} = \frac{13}{2}\{2\times 2 + 12\times\frac{1}{2}\} = \frac{13}{2}\{10\} = \boxed{65}$ Ans. 2

(ii) $l_3 = \frac{2}{3}$, $l_1 + l_2 = 2\frac{1}{2}$, $a = ?$, $r = ?$, $l_4 = ?$

$\therefore ar^2 = \frac{2}{3}$ and $a + ar = \frac{5}{2}$ or $ar^2 = \frac{2}{3} \ \ldots\ldots (1)$, $a(1+r) = \frac{5}{2} \ \ldots\ldots (2)$

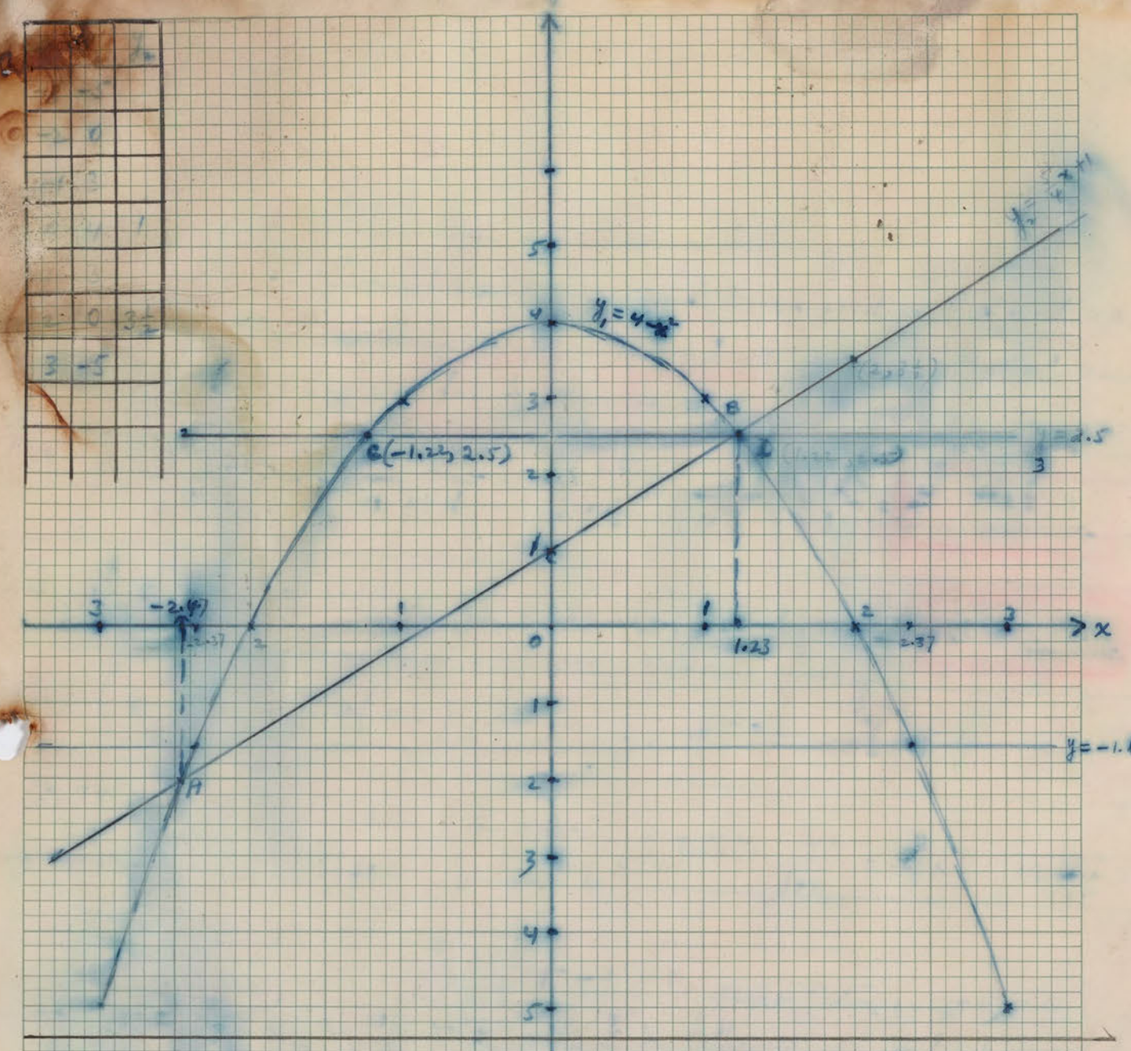
by Division $\dfrac{r^2}{1+r} = \dfrac{2}{3}\times\dfrac{2}{5}$ or $\dfrac{r^2}{1+r} = \dfrac{4}{15}$

$\therefore 4+4r = 15r^2$ or $15r^2 - 4r - 4 = 0$ or $(5r+2)(3r-2) = 0$

or $r = \frac{2}{3}$ and $r = -\frac{2}{5}$ {the latter value is to be discarded since all the terms of the progression are positive, given}

$\therefore$ from eq. (1), $a\left(\frac{2}{3}\right)^2 = \frac{2}{3} \quad \therefore \frac{2}{3}a = 1 \quad \therefore a = \frac{3}{2} \quad \therefore l_4 = ar^3 = \frac{3}{2}\left(\frac{2}{3}\right)^3 = \frac{4}{9}$

$\boxed{a = \frac{3}{2},\ r = \frac{2}{3},\ l_4 = \frac{4}{9}}$ Ans.



(i) From the table above, the graph of $y_1 = 4 - x^2$ is plotted as shown. Also the st. line $y_2 = \frac{5}{4}x + 1$

(ii) The two graphs intersect at two points whose abscissas are : $x = -2.47$ and $x = 1.23$ Ans.

$\therefore 4 - x^2 > \frac{5}{4}x + 1$ between -2.47 and 1.23 since between these two values of x the curve of $4 - x^2$ lies above the st. line $\frac{5}{4}x + 1$

(iii) (b) We draw the line $y = 2.5$. This line cuts the graph at two points C$(-1.22, 2.5)$ and D$(1.22, 2.5)$

$\therefore 4 - x^2 = 2.5$ at $x = -1.22$ and $x = 1.22$ Ans.

(iii) (c) From $y = 4 - x^2$, we get $x^2 = 4 - y$

or $x = \pm\sqrt{4 - y}$. Now let $4 - y = 5.6$

$\therefore y = -1.6$ $\therefore$ when $y = -1.6$, $x = \pm\sqrt{4 - (-1.6)}$ or $x = \pm\sqrt{5.6}$ and from the graph we find that when $y = -1.6$, $x = \pm 2.37$ Ans.

Final Exam. paper in Algebra

18/5/1967

$\therefore xt = at + b \therefore t(x-a) = b \therefore t = \frac{b}{x-a}$

$\therefore y = \frac{bx}{x-a}$ Ans.

(ii) $2^{x-y} = 8 \therefore 2^{x-y} = 2^3 \therefore x-y = 3$

$3^{x+y} = 9 = 3^2 \therefore x+y = 2$

$\therefore x = \frac{5}{2}$, $y = -\frac{1}{2}$ Ans. (7 marks)

(iii) $\frac{m}{n} = \frac{5}{4}$, $\frac{p}{q} = \frac{3}{4}$, $\frac{4m+5p}{n+q} = 5$

$\therefore m = \frac{5n}{4}$, $\therefore p = \frac{3q}{4}$ $\therefore \frac{4m+5p}{n+q} = \frac{15n}{4} + \frac{15q}{4} = \frac{15}{4}$

$\therefore 3m = \frac{15n}{4}$, $\therefore 3p = \frac{15q}{4}$

$\therefore \frac{3m+3p}{n+q} = \frac{15}{4}$ Ans. (7

2. (a) When n tables are made, cost of table cloth: $\pounds\left(\frac{300}{n}+8\right)n$

(b) When $(50+n)$ tables are made, cost of table cloth: $\pounds\left[\frac{300+8(50+n)}{50+n}\right](50+n)$

or 1 table costs: $\pounds\left(\frac{300}{50+n}+8\right)$

$\therefore \frac{300}{50+n} + 8 + 1 = \frac{300}{n} + 8$

or $\frac{300}{n} - \frac{300}{50+n} = 1$ or $300(50+n) - 300n = n(50+n)$

or $15000 + 300n - 300n = n^2 + 50n$ or $n^2 + 50n - 15000 = 0$

$\therefore (n+150)(n-100) = 0$ $\therefore n = -150$ (to be discarded)

or $n = 100$ Ans.

$\therefore$ Cost of one table in case (a) is $\pounds\left(\frac{300}{n}+8\right) = \pounds(3+8) = \pounds 11$

$\therefore$ " " " " " (b) is $\pounds\left(\frac{300}{50+n}+8\right) = \pounds(3+8) = \pounds 10$ Ans.

(20 marks)

$$x = \sqrt{\frac{\cos^3 42°17' \times \tan^5 27°34'}{1.009^3 \times 90.04^5}}$$

$\log \cos 42°17'$	$= \bar{1}.8691$	$3 \log \cos 42°17' = \bar{1}.6073$
$\log \tan 27°34'$	$= \bar{1}.7177$	$5 \log \tan 27°34' = \bar{2}.5885$
$\log 1.009$	$= 0.0037$	$\log$ Num. $= \bar{2}.1958$
$\log 90.04$	$= 1.9544$	$\log$ Den. $= 9.7831$
$3 \log 1.009$	$= 0.0111$	$7 \log x = \bar{12}.4127$
$5 \log 90.04$	$= 9.7720$	$\log x = \bar{2}.34467$
$\log$ Den.	$= 9.7831$	or $\log x = \bar{2}.3447$
		$x = 0.02212$
		or $x = 2.212 \times 10^{-2}$.

Ans.

(10 marks)

(ii) $N = 2^{4.136}$ and $10^{0.301} = 2$, $\log_{10} N = ?$

$\log_{10} N = 4.136 \log_{10} 2$ but $\log_{10} 2 = 0.301$

$\therefore \log_{10} N = 4.136 \times 0.301 = \bar{1}.244936 = 1.2449$ (Correct to 4 significant figures)

Ans.

(10 marks)

(i) 4, 12, 20, 28, ...

In this progression $a = 4$, $d = 8$

$\therefore S = \frac{n}{2}\{2\times 4 + (n-1)\times 8\}$ or $S = \frac{n}{2}\{8 + 8n - 8\}$ or

$S = \frac{n}{2}(8n)$ $\therefore S = 4n^2$ or $S = (2n)^2$ Ans.

$\therefore$ Whatever the value of n is, the sum S is always a perfect square

Q.E.D.

(10 marks)

(ii) 12, x, y, 4 ... from the statement of the question:

12, x, y are in A.P. $\therefore x - 12 = y - x$ ..... ①

also x, y, 4 are in G.P $\therefore \frac{y}{x} = \frac{4}{y}$ ..... ②

from eq. ①: $y = 2x - 12$ ----- ③ $\therefore y^2 = (2x-12)^2 = 4(x-6)^2$

from eq. ②: $y^2 = 4x$ ---- ④ or $y^2 = 4(x^2 - 12x + 36)$

$\therefore 4(x^2 - 12x + 36) = 4x$ $\therefore x^2 - 13x + 36 = 0$

$\therefore (x-4)(x-9) = 0$ $\therefore x = 4$ or $x = 9$

when $x = 4$, $y = 2(x-6) = 2(4-6) = -4$

when $x = 9$, $y = 2(x-6) = 2(9-6) = 6$

$\therefore$ $x = 4$, $y = -4$ } Ans. 1 $\quad$ $x = 9$, $y = 6$ } Ans. 2

(10 marks)

$\therefore$ the terms are either 12, 4, -4, 4

or 12, 9, 6, 4

4th year Final Exam. 18/5/1961 Algebra

x	x^3	y_1
-2	-8	-8
-1	-1	-3
0	0	2
1	1	
2	8	12
3	27	

(6 marks)

(i) The curve $y_1 = x^3$ is plotted as shown below

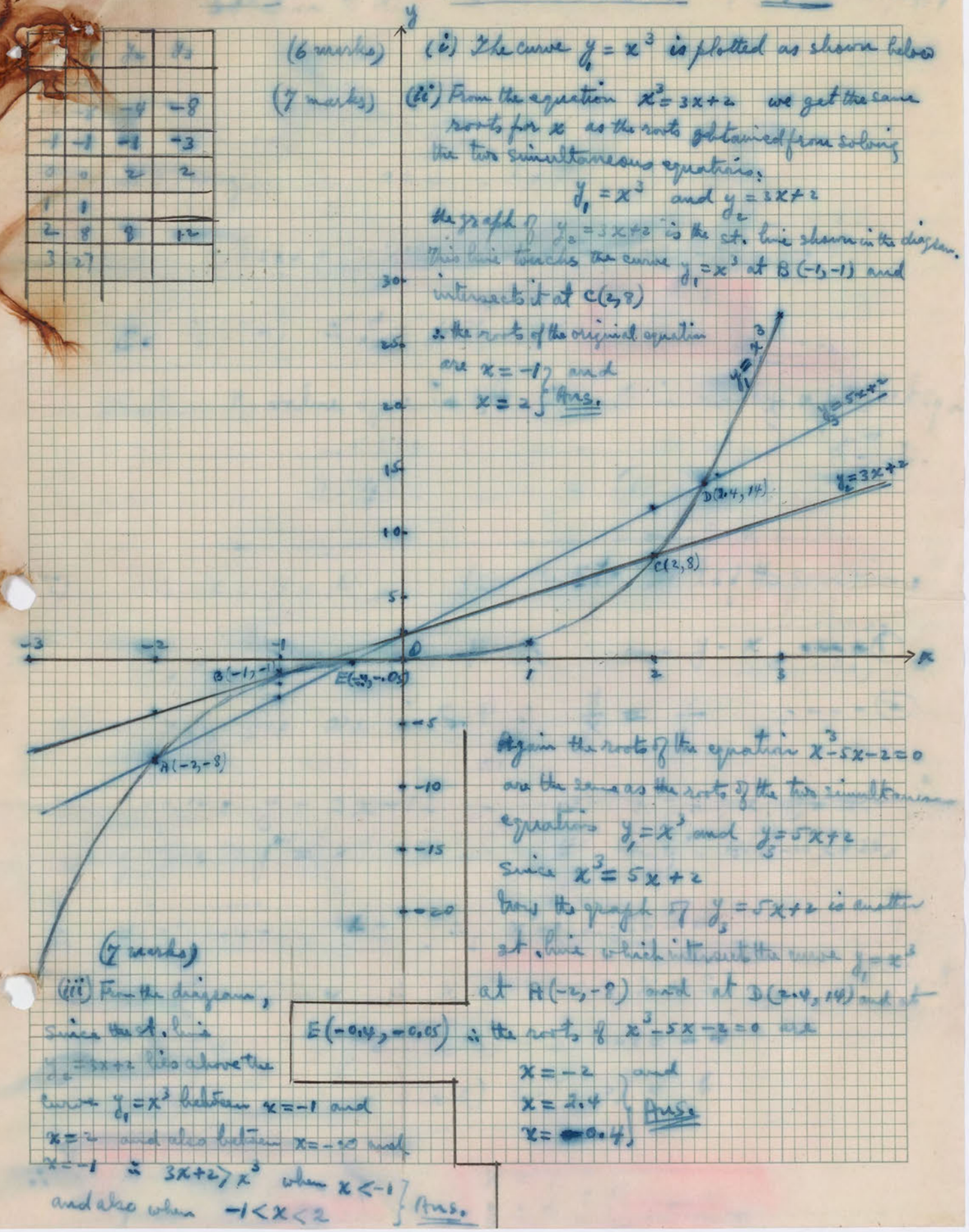
(7 marks)

(ii) From the equation $x^3 = 3x + 2$ we get the same roots for x as the roots obtained from solving the two simultaneous equations:

$y_1 = x^3$ and $y_2 = 3x + 2$

the graph of $y_2 = 3x + 2$ is the st. line shown in the diagram. This line touches the curve $y_1 = x^3$ at B(-1,-1) and intersects it at C(2,8)

∴ the roots of the original equation are $x = -1$ and $x = 2$ } Ans.



Again the roots of the equation $x^3 - 5x - 2 = 0$ are the same as the roots of the two simultaneous equations $y_1 = x^3$ and $y_3 = 5x + 2$

Since $x^3 = 5x + 2$

here the graph of $y_3 = 5x + 2$ is another st. line which intersects the curve $y_1 = x^3$ at A(-2,-8) and at D(2.4, 14) and at E(-0.4, -0.05) ∴ the roots of $x^3 - 5x - 2 = 0$ are

$x = -2$ and
$x = 2.4$
$x = -0.4$ } Ans.

(7 marks)

(iii) From the diagram, since the st. line $y_2 = 3x + 2$ lies above the curve $y_1 = x^3$ between $x = -1$ and $x = 2$ and also between $x = -\infty$ and $x = -1$ ∴ $3x + 2 > x^3$ when $x < -1$ and also when $-1 < x < 2$ } Ans.

Shamash Secondary School
Final Examination, May, 1967

Subject: Algebra
Class: 4th Year Secondary

Date: 18/5/1967
Time: 8:00 - 11:00 a.m.

Answer all questions:

1. (i) If $x = a + \frac{b}{t}$ and $y = b + at$, find an expression for y in terms of a, b, x. (6 marks)

(ii) If $2^{x-y} = 8$ and $3^{x-2y} = 9$ find the values of x and y. (7 marks)

(iii) If $\frac{m}{n} = \frac{5}{4}$ and $\frac{p}{q} = \frac{3}{4}$, find the value of $\frac{3m+5p}{n+q}$ (7 marks)

2. A manufacturer calculated that, allowing for the initial outlay on equipment, the cost of production of n tables was £(300+8n). Write down the cost of production of 1 table when

(a) n tables are made,

(b) (50+n) tables are made.

If the cost of production of one table decreases by £1 when the extra 50 tables are made, find the value of n and the cost of production of a table in each case.

(20 marks)

3. (i) Compute by logarithms the following expression, arranging your work neatly:

$$\sqrt[7]{\frac{\cos^3 42^\circ 17' \times \tan^5 27^\circ 34'}{1.009^3 \times 90.04^5}}$$

(10 marks)

(ii) If $N = 2^{4.136}$ and $10^{0.301} = 2$, find, without the use of tables, the logarithm of N to the base 10 correct to five significant figures.

(10 marks)

4. (i) Prove that the sum of any number of terms of the progression 4, 12, 20, 28, ... is a perfect square.

(ii) The first three of the four terms, 12, x, y, 4, are in arithmetical progression and the last three are in geometrical progression. Find x and y.

(p.2)..

Algebra. 4th Year. 18/5/67

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5. (i) Draw the graph of $y = x^3$ from $x = -3$ to $x = +3$, using 1 inch for 1 unit on the x-axis and 1 inch for 10 units on the y-axis.

(6 marks).

(ii) By drawing two straight-line graphs on the same diagram find the roots of each of the following equations correct to one decimal place.

$x^3 = 3x+2$ .....(1)

$x^3-5x-2=0$ .....(2)

(7 marks)

(iii) Find from the resulting diagram the range of values of x for which $(3x+2)$ is greater than x^3.

(7 marks).

Solution of 4th Quarter Exam. May 1967.

4th year Scientific

1.

$\log 0.4007 = \bar{1}.6029$	$3 \log 0.4007 = \bar{3}.8087$
$\log \tan 31°19' = \bar{1}.7821$	$2 \log \tan 31°19' = \bar{1}.7642$
$\log 50.72 = 1.7052$	$\log$ Num. $= \bar{3}.5729$
$\log \cos 14°34' = \bar{1}.9858$	$\log$ Den. $= 8.4834$
$5 \log 50.72 = 8.5260$	$9 \log x = \bar{10}.0895$
$2 \log \cos 14°34' = \bar{1}.9574$	$\log x = \bar{2}.8988$
$\log$ Den. $= 8.4834$	$x = 0.07921$
	$= 7.921 \times 10^{-2}$ Ans.

$$x = \sqrt[9]{\frac{(0.4007)^3 \times \tan^2 31°19'}{50.72^5 \times \cos^2 14°34'}}$$

2. (i) Simplify:

$$\left(x^{1+\frac{1}{p}}\right)^{\frac{p}{p+q}} \div \sqrt[p]{\frac{x^{2p}}{(x^{-1})^{-p}}} = x^{\frac{p}{p+q}+\frac{q}{p+q}} \div \sqrt[p]{x^{2p} \cdot x^{-p}}$$

$$= x^{\frac{p+q}{p+q}} \div x^{\frac{p}{p}} = x \div x = 1 \quad \text{Ans.}$$

2(ii) Simplify :

$$\left\{\frac{y^{\frac{1}{2}}+y^{-\frac{1}{2}}}{y^2-y+1}-\frac{y^{\frac{1}{2}}-y^{-\frac{1}{2}}}{y^2+y+1}\right\}\div\left\{\frac{y^{\frac{1}{2}}+2y^{-\frac{1}{2}}}{y^3-1}-\frac{y^{\frac{1}{2}}-2y^{-\frac{1}{2}}}{y^3+1}\right\}$$

$$\left\{\frac{y+1}{y^{\frac{1}{2}}(y^2-y+1)}-\frac{(y-1)}{y^{\frac{1}{2}}(y^2+y+1)}\right\}\div\left\{\frac{y+2}{y^{\frac{1}{2}}(y^3-1)}-\frac{y-2}{y^{\frac{1}{2}}(y^3+1)}\right\}$$

$$\frac{(y+1)(y^2+y+1)-(y-1)(y^2-y+1)}{y^{\frac{1}{2}}(y^2-y+1)(y^2+y+1)}\div\frac{(y+2)(y^3+1)-(y-2)(y^3-1)}{y^{\frac{1}{2}}(y^3-1)(y^3+1)}$$

$$=\frac{y^3+y^2+y+y^2+y+1-(y^3-y^2+y-y^2+y-1)}{y^{\frac{1}{2}}(y^2-y+1)(y^2+y+1)}\times\frac{y^{\frac{1}{2}}(y^3-1)(y^3+1)}{y^5+y+2y^3+2-(y^5-y-2y^3+2)}$$

$$=\frac{4y^2+2}{y^{\frac{1}{2}}(y^2-y+1)(y^2+y+1)}\times\frac{y^{\frac{1}{2}}(y^3-1)(y^3+1)}{4y^3+2y}=$$

$$=\frac{2(2y^2+1)}{y^{\frac{1}{2}}(y^2-y+1)(y^2+y+1)}\times\frac{y^{\frac{1}{2}}(y-1)(y^2+y+1)(y+1)(y^2-y+1)}{2y(2y^2+1)}$$

$$=\frac{(y-1)(y+1)}{y}=\frac{y^2-1}{y}=y-\frac{1}{y}\quad\text{Ans.}$$

$2 \times 3^{2x} = 4^{x-1}$

$\log 2 + 2x \log 3 = (x-1)\log 4$

$2x \log 3 - x \log 4 = -\log 4 - \log 2$

$2x \log 3 - 2x \log 2 = -2\log 2 - \log 2$

$2x(\log 3 - \log 2) = -3 \log 2$

$$x = -\frac{3\log 2}{2(\log 3 - \log 2)} = -\frac{3 \times 0.3010}{2(0.4771 - 0.3010)}$$

$$\therefore x = -\frac{3 \times 0.3010}{2 \times 0.1761} = -\frac{0.9030}{0.3522} = -2.5638\ldots$$

$\therefore x = -2.564$ Correct to four significant figures.

Shamash Secondary School
4th Quarter Exam. May 5th, 1967

Subject: Mathematics
Class: 4th Year Secondary

Date: 7/5/1967
Time: 8:00 - 9:30

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1. Compute by logarithm the expression:

$$\sqrt[9]{\frac{(0.4007)^3 \times \tan^2 37^\circ 19'}{50.72^5 \times \cos^3 14^\circ 34'}}$$

(30 marks)

2. Simplify: (i) $\left(x^{1+\frac{q}{p}}\right)^{\frac{p}{p+q}} \div \sqrt[p]{\frac{2p}{x}}{(x^{-1})^{-p}}$

(20 marks)

(ii) $\left\{\frac{y^{\frac{1}{2}} + y^{-\frac{1}{2}}}{y^2-y+1} - \frac{y^{\frac{1}{2}} - y^{-\frac{1}{2}}}{y^2+y+1}\right\} \div \left\{\frac{y^{\frac{1}{2}}+2y^{-\frac{1}{2}}}{y^3-1} - \frac{y^{\frac{1}{2}}-2y^{-\frac{1}{2}}}{y^3+1}\right\}$

(20 marks)

3. Solve the equation: $2x3^{2x} = 4^{x-1}$ finding the answer correct to four significant figures.

(30 marks)

Shamash Secondary School
4th Quarter Exam. May 5th, 1967

Subject: Mathematics
Class: 4th Year Secondary

Date: 7/5/1967
Time: 8:00 - 9:30

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1. Compute by logarithm the expression:

$$\sqrt[9]{\frac{(0.4007)^3 \times \tan^2 37^\circ 19'}{50.72^5 \times \cos^3 14^\circ 34'}}$$

(30 marks)

2. Simplify: (i) $\left(x^{1+\frac{q}{p}}\right)^{\frac{p}{p+q}} \div \sqrt[p]{\frac{x^{2p}}{(x^{-1})^{-p}}}$

(20 marks)

(ii) $\left\{\frac{y^{\frac{1}{2}} + y^{-\frac{1}{2}}}{y^2-y+1} - \frac{y^{\frac{1}{2}} - y^{-\frac{1}{2}}}{y^2+y+1}\right\} \div \left\{\frac{y^{\frac{1}{2}}+2y^{-\frac{1}{2}}}{y^3-1} - \frac{y^{\frac{1}{2}}-2y^{-\frac{1}{2}}}{y^3+1}\right\}$

(20 marks)

3. Solve the equation: $2x3^{2x} = 4^{x-1}$ finding the answer correct to four significant figures.

(30 marks)

Solutions to 3rd Quarter Exam. in Algebra

10th Year Secondary

20/3/1967

2. (i) $\frac{b}{\sqrt{a}} \cdot \sqrt[3]{ac} \cdot \frac{\sqrt[4]{c^3}}{\sqrt{b}} \cdot \frac{\sqrt{b^{-1}}}{a^{-\frac{1}{6}}} = \frac{b}{a^{\frac{1}{2}}} \cdot a^{\frac{1}{3}} \cdot c^{\frac{1}{3}} \cdot \frac{c^{\frac{3}{4}}}{b^{\frac{1}{2}}} \cdot \frac{b^{-\frac{1}{2}}}{a^{-\frac{1}{6}}}$

$= a^{-\frac{1}{2}+\frac{1}{3}+\frac{1}{6}} \cdot b^{1-\frac{1}{2}-\frac{1}{2}} \cdot c^{\frac{1}{3}+\frac{3}{4}} = a^{\frac{-3+2+1}{6}} \cdot b^{\frac{2-1-1}{2}} \cdot c^{\frac{4+3}{12}}$

$= a^0 \cdot b^0 \, c^{\frac{13}{12}} = \sqrt[12]{c^{13}} = c\sqrt[12]{c}$ Ans.

(ii) $\left[\frac{(9^{n+\frac{1}{2}})(\sqrt{3 \cdot 3^n})}{3\sqrt{3^{-n}}}\right]^{\frac{1}{n}} = \left[\frac{(3^2)^{n+\frac{1}{2}} \cdot (3^{\frac{n+n}{2}})^{\frac{1}{2}}}{3 \cdot 3^{-\frac{n}{2}}}\right]^{\frac{1}{n}} = \left[3^{2n+\frac{1}{2}} \cdot 3^{\frac{n+1}{2}} \cdot 3^{\frac{n}{2}-1}\right]^{\frac{1}{n}}$

$= \left[3^{\frac{4n+1+n+1+n-2}{2}}\right]^{\frac{1}{n}} = \left[3^{\frac{6n}{2}}\right]^{\frac{1}{n}} = 3^{\frac{3n}{n}} = 3^3 = 27$ Ans.

3. $\left(\sqrt[3]{x^2} + 2x^{\frac{1}{3}} - 16x^{-\frac{2}{3}} - \frac{32}{x}\right) \div \left(x^{\frac{1}{6}} + 4x^{-\frac{1}{6}} + \frac{4}{\sqrt{x}}\right)$

$\left(x^{\frac{2}{3}} + 2x^{\frac{1}{3}} - 16x^{-\frac{2}{3}} - 32x^{-1}\right) \div \left(x^{\frac{1}{6}} + 4x^{-\frac{1}{6}} + 4x^{-\frac{1}{2}}\right)$

$x^{\frac{1}{6}} + 4x^{-\frac{1}{6}} + 4x^{-\frac{1}{2}}$ | $x^{\frac{2}{3}} + 2x^{\frac{1}{3}} - 16x^{-\frac{2}{3}} - 32x^{-1}$ | $x^{\frac{1}{2}} - 2x^{\frac{1}{6}} + 4x^{-\frac{1}{6}} - 8x^{-\frac{1}{2}}$

$x^{\frac{2}{3}} + 4x^{\frac{1}{3}} + 4x^0$

$-2x^{\frac{1}{3}} - 4 - 16x^{-\frac{2}{3}} - 32x^{-1}$

$-2x^{\frac{1}{3}} - 8 - 8x^{-\frac{1}{3}}$

$4 + 8x^{-\frac{1}{3}} - 16x^{-\frac{2}{3}} - 32x^{-1}$

$4 + 16x^{-\frac{1}{3}} + 16x^{-\frac{2}{3}}$

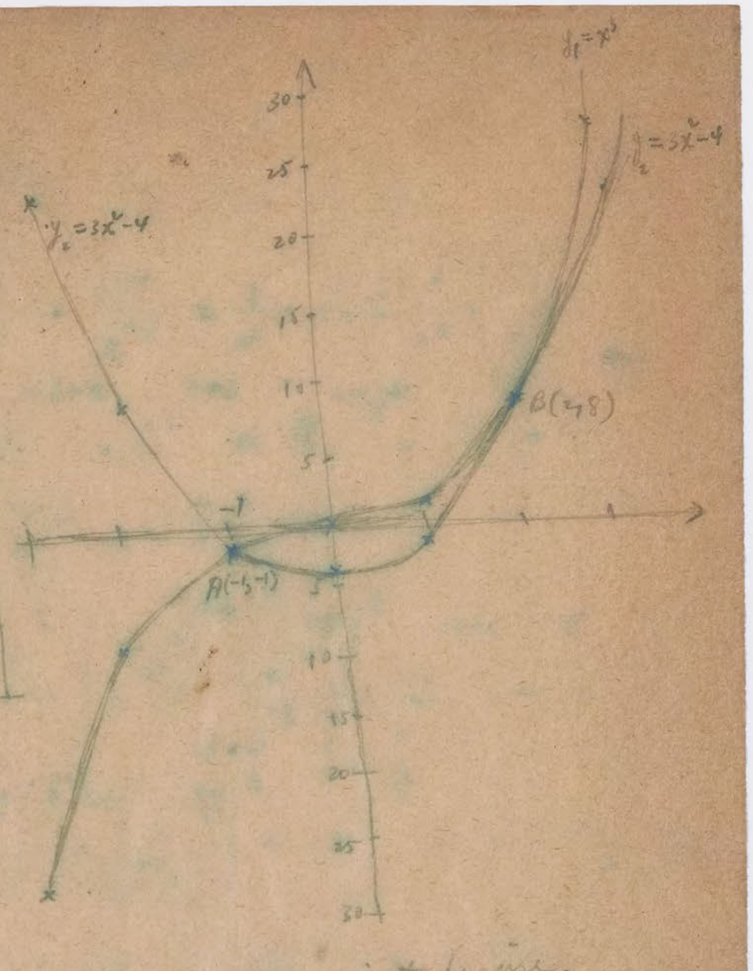
$-8x^{-\frac{1}{3}} - 32x^{-\frac{2}{3}} - 32x^{-1}$

$-8x^{-\frac{1}{3}} - 32x^{-\frac{2}{3}} - 32x^{-1}$

$y_1 = x^3$

$y_2 = 3x^2 - 4$

x	$y_1 = x^3$	$y_2 = 3x^2 - 4$
-3	-27	23
-2	-8	8
-1	-1	-1
0	0	-4
1	1	-1
2	8	8
3	27	23



(i) The two curves are plotted as they appear in the figure.

(ii) The roots of the equation $x^3 - 3x^2 + 4 = 0$ are the same as the roots of the equation $x^3 = 3x^2 - 4$. These roots are the same as the abscissas of the points of intersections of the two curves $y_1 = x^3$ and $y_2 = 3x^2 - 4$, which are $x = -1$ and $x = 2$. Since the two curves have points $A(-1, -1)$ & $B(2, 8)$ as common points between them.

(iii) (a) The expression $x^3 - 3x^2 + 4$ is always negative when $x^3 < 3x^2 - 4$ or when $y_1 < y_2$. But $y_1 < y_2$ for all values of $x < -1$ since the curve $y_1 = x^3$ lies below the curve $y_2 = 3x^2 - 4$.

(b) Also the expression $x^3 - 3x^2 + 4$ is always positive when $y_1 > y_2$ (except $x = 2$) and this is the case for all values of $x > -1$, since for all these values of x, the curve y_1 lies above the curve y_2. Q.E.D.

Shamash Secondary School
3rd Quarter Examination, March 1967.

Subject: Algebra
Class: 4th Secondary

Date: 20/3/1967
Time: 8:30 - 10:00 a.m.

1- (i) Draw the graphs of $y = x^3$ and $y = 3x^2-4$ on the same axes, for values of x from $x = -3$ to $x = 3$

(20 marks)

(ii) From your graphs find the roots of the equation $x^3-3x^2+4=0$

(15 marks).

(iii) For what values of x is the expression x^3-3x^2+4 always negative? always positive ? Use your graphs to explain why.

(15 marks)

2- Simplify (i) $\frac{b}{\sqrt{a}} \cdot \sqrt[3]{ac} \cdot \frac{\sqrt[4]{c^3}}{\sqrt{b}} \cdot \frac{\sqrt{b^{-1}}}{a^{-\frac{1}{6}}}$

(15 marks)

(ii) $\left[\frac{(9^{n+\frac{1}{4}}) \cdot \sqrt{(3)(3^n)}}{3\sqrt{3^{-n}}}\right]^{\frac{1}{n}}$

(15 marks)

3- Divide $(\sqrt[3]{x^2} + 2x^{\frac{1}{3}} - 16x^{-\frac{2}{3}} - \frac{32}{x})$ by $(x^{\frac{1}{6}} + 4x^{-\frac{1}{6}} + \frac{4}{\sqrt{x}})$

(20 marks)

Shamash Secondary School
Mid-Year Examination, Feb.1967

Subject: Algebra
Class: 4th Year, Secondary

Date: 6/2/1967
Time: 8:30 - 11:30 a.m.

Attempt all questions:

1. Revolve into factors:

   (i) $3x^2-(4a+2b)x+a^2+2ab$ (6 marks)

   (ii) $8x^3-27y^3+z^3+18xyz$ (6 ")

   (iii) Divide $(a^2+b^2+c^2)(a+1)+(2ab-2ac)(a+1)-2abc-2bc$ by $(a+1)$ and express the quotient as a perfect square. (8 marks)

2. (i) If $x+y = 2a$ and $x-y=2b$, find in the shortest possible way, the value of $x^4+x^2y^2+y^4$. (10 marks)

   (ii) Find the value of p which will make the expression $2x^3+px^2-5x+2$ divisible by $(x+2)$ and find the other two factors.
   (10 marks)

3. (i) A man can row upstream at 'a' miles an hour and downstream at 'b' miles an hour. He rows up to a certain point and then returns to his starting point, and finds that his average speed is 's' miles an hour for the double journey. Express each of the letters in terms of the other two. Find the value of 'b' if a=2 and s=3.
   (10 marks)

   (ii) Solve the two simultaneous equations:

   $x^2+4y^2+80 = 15x+30y$ .....(1)

   $xy = 6$ . .....(2)
   (10 marks)

4. Two men started at the same time to meet each other from points which were 26 miles apart. If one took 4½ minutes longer than the other to walk a mile, and they met 2 hours after starting, find the speed of each in miles per hour.
   (20 marks)

(cont'd.p.2)...

Algebra. 4th Secondary. 6/2/1967

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5. (i) Draw on the same diagram the graphs of the function $4x-3$, and of the function $4x^2-4x-15$, taking ½ inch as one unit on the x-axis and one tenth of an inch as one unit on the y-axis. (8 marks)

(ii) From your diagram, find the roots of the two simultaneous equations $y_1 = 4x-3$ .....(1)

$y_2 = 4x^2-4x-15$ ....(2) (7 marks)

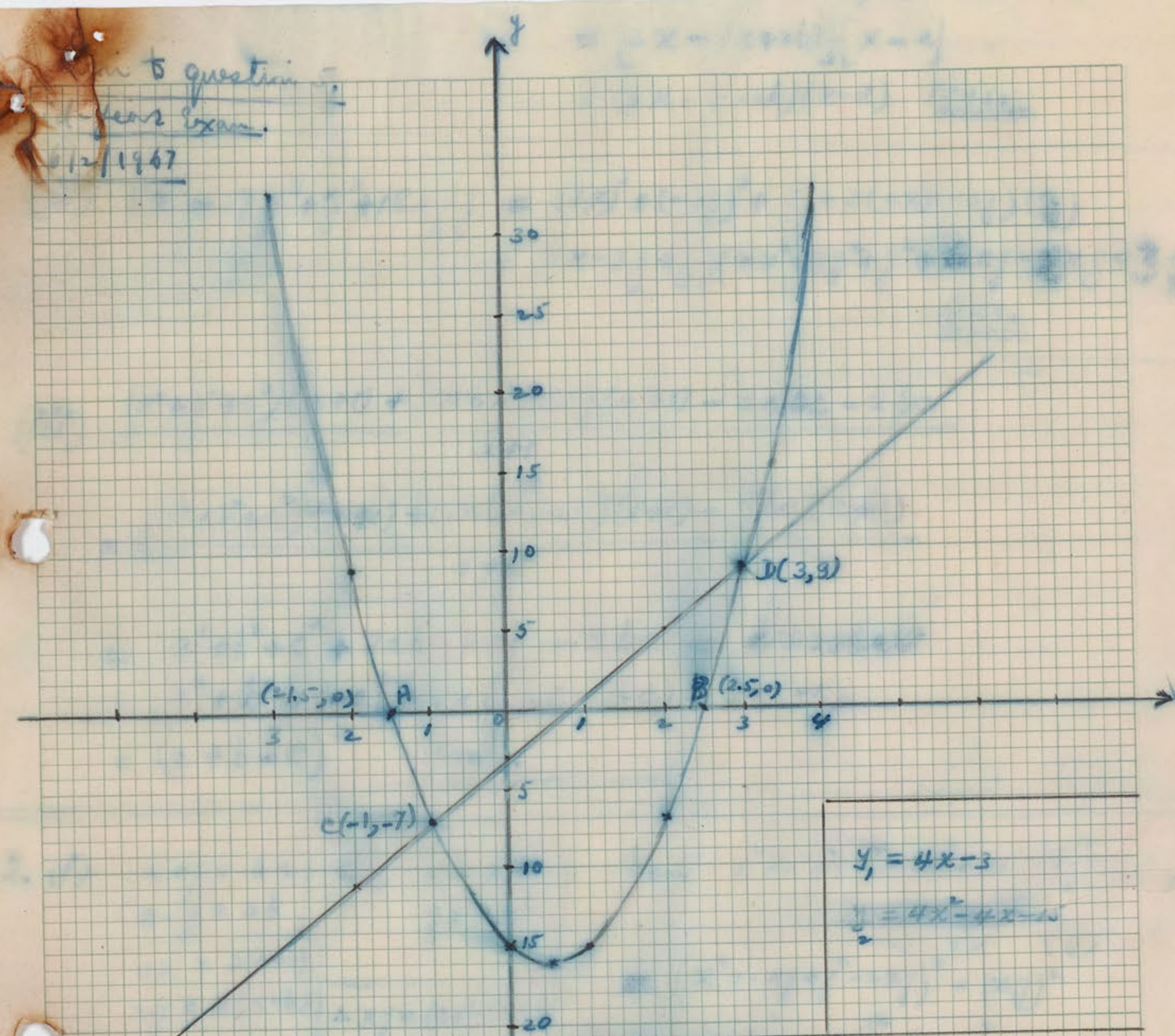
(iii) From the graph of the function $4x^2-4x-15$, find the roots of the equation $4x^2-4x-15=0$.

(5 marks)

---

	x	y_1	y_2
	-2	-11	9
	-1		-7
	0	-3	-15
	$\frac{1}{2}$		-16
	1		-15
	2	5	-7
	3		9
	4		33

… to question …
… Years Exam.
6/12/1967



x	y_1	y_2
-3		33
-2	-11	9
-1		-7
0	-3	-15
$\frac{1}{2}$		-16
1		-15
2	5	-7
3		9
4		33

(i) The two graphs are drawn as shown above.

(ii) The roots of the two simultaneous equations are the coordinates of the two points of intersection C(-1, -7) and D(3, 9). (i.e.)

$\left.\begin{array}{l} x = -1 \\ y = -7 \end{array}\right\}$ Ans. 1 $\quad$ $\left.\begin{array}{l} x = 3 \\ y = 9 \end{array}\right\}$ Ans. 2

(iii) The roots of the equation $4x^2 - 4x - 15 = 0$ are the ~~abscissas~~ abscissas of the points of intersection of the curve with the x-axis, i.e.:

$x = -1.5$ Ans. 1

$x = 2.5$ Ans. 2

Solution to Final Exam. Questions to 4th year in Algebra Cuba

February 6th 1967

1. (i) $3x^2-(4a+2b)x+a^2+2ab = 3x^2-(4a+2b)x+a(a+2b)$

$= [3x-(a+2b)][x-a]$

$= (3x-a-2b)(x-a)$ Ans.

(ii) $8x^3-27y^3+z^3+18xyz = (2x)^3+(-3y)^3+z^3-3(2x)(-3y)(z)$

$= (2x-3y+z)(4x^2+9y^2+z^2+6xy-2xz+3yz)$

Ans.

(iii) $\dfrac{(a^2+b^2+c^2)(a+1)+(2ab-2ac)(a+1)-2abc-2bc}{a+1}$

$= \dfrac{(a^2+b^2+c^2)(a+1)+(2ab-2ac)(a+1)-2bc(a+1)}{a+1}$

$= a^2+b^2+c^2+2ab-2ac-2bc =$

$= a^2+b^2+(-c)^2+2ab+2a(-c)+2b(-c)$

$= (a+b-c)^2$ Ans.

2. (i) $\begin{cases} x+y=2a \\ x-y=2b \end{cases}$ or $x=a+b$, $y=a-b$

$2x=2(a+b)$

$2y=2(a-b)$

$\therefore xy=(a+b)(a-b)$

or $xy=a^2-b^2$

Now $x^4+x^2y^2+y^4 = x^4+2x^2y^2+y^4-x^2y^2$

$= (x^2+y^2)^2-(xy)^2$

$= (x^2+2xy+y^2-2xy)^2-(xy)^2$

$= [(x+y)^2-2xy]^2-(xy)^2$

$= [(x+y)^2-2xy+xy][(x+y)^2-2xy-xy] = [(x+y)^2-xy][(x+y)^2-3xy]$

$= [(2a)^2-(a^2-b^2)][(2a)^2-3(a^2-b^2)] = (4a^2-a^2+b^2)(4a^2-3a^2+3b^2)$

$= (3a^2+b^2)(a^2+3b^2)$ Ans.

$(2x^3+px^2-5x+2)\div(x+2) = 2x^2+(p-4)x-2p+3+\frac{4p-4}{x+2}$

$2x^3+px^2-5x+2 \;\big|\; 2x^2+(p-4)x-2p+3$

$2x^3+4x^2$

$(p-4)x^2-5x+2$

$(p-4)x^2+2(p-4)x$

$(-2p+3)x+2$

$(-2p+3)x-4p+6$

$4p-4$ = Remainder $=0$

$\therefore 4p-4=0 \quad \therefore p=1$

$\therefore$ Original expression $=$

$2x^3+x^2-5x+2 =$

$= [x+2][2x^2+(p-4)x-2p+3]$

$= (x+2)(2x^2-3x+1) =$

$= (x+2)(2x-1)(x-1)$ ■

$\therefore p=1$ and the other two factors are $(2x-1)$ and $(x-1)$ Ans.

(i) Rate of rowing upstream = a m.p.h.
" " " downstream = b m.p.h.
} let the distance rowed each way = x miles

$\therefore \frac{x}{a}+\frac{x}{b}=\frac{2x}{s}$ or

$\frac{1}{a}+\frac{1}{b}=\frac{2}{s} \;\therefore\; bs+as=2ab$ or $2ab-as=bs$ or $a(2b-s)=bs$

or $a=\frac{bs}{2b-s}$ Ans. 1

also $2ba-bs=as \;\therefore\; b(2a-s)=as \;\therefore\; b=\frac{as}{2a-s}$ Ans. 2

also $bs+as=2ab \;\therefore\; s(a+b)=2ab \;\therefore\; s=\frac{2ab}{a+b}$ Ans. 3.

but $b=\frac{as}{2a-s}=\frac{2\times3}{2\times2-3}=\frac{6}{1}=6$ Ans. 4

(ii) $\left.\begin{aligned} x^2+4y^2+80 &= 15x+30y \quad \cdots\cdots ① \\ xy &= 6 \quad \cdots\cdots ② \end{aligned}\right\}$ $\therefore$ from ②: $4xy=24 \quad \cdots\cdots ③$

add eq. ① + ③ and you get:

$x^2+4xy+y^2+80=15x+30y+24$ or $(x+2y)^2+80=15(x+2y)+24$ or

$(x+2y)^2-15(x+2y)+56=0 \;\therefore\; [(x+2y)-7][(x+2y)-8]=0$ or

$x+2y-7=0$ or $x=7-2y \quad \cdots\cdots ④$ also

$x+2y-8=0$ or $x=8-2y \quad \cdots\cdots ⑤$

from eq. ② + ④ $(7-2y)y=6$ or $7y-2y^2=6 \;\therefore\; 2y^2-7y+6=0 \;\therefore (2y-3)(y-2)=0$

$\therefore y=2$ and $y=\frac{3}{2}$ $\therefore$ from ④: $x=7-4=3$ and $x=7-3=4$

$\therefore \left.\begin{matrix} x=3 \\ y=2 \end{matrix}\right\}$ Ans. $\left.\begin{matrix} x=4 \\ y=\frac{3}{2} \end{matrix}\right\}$ Ans.

again from ② + ⑤: $(8-2y)y=6 \;\therefore\; 2y^2-8y+6=0$

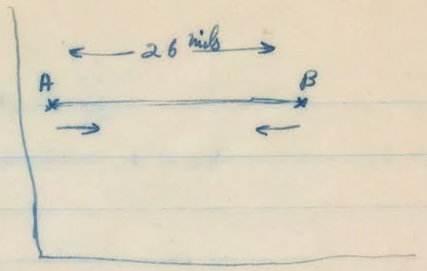
$\therefore y^2-4y+3=0 \;\therefore (y-1)(y-3)=0 \;\therefore y=1, y=3$

from ⑤: $x=8-2=6$ and $x=8-6=2$ $\therefore \left.\begin{matrix} x=6 \\ y=1 \end{matrix}\right\}$ Ans. $\left.\begin{matrix} x=2 \\ y=3 \end{matrix}\right\}$ Ans.

$\therefore \left.\begin{matrix} x=2 \\ y=3 \end{matrix}\right\}$ Ans. 1 $\left.\begin{matrix} x=3 \\ y=2 \end{matrix}\right\}$ Ans. 2 $\left.\begin{matrix} x=6 \\ y=1 \end{matrix}\right\}$ Ans. 3 $\left.\begin{matrix} x=4 \\ y=\frac{3}{2} \end{matrix}\right\}$ Ans. 4

Solution to Final Exam. Question in Algebra to 4th year Cont.

February 6th 1967.



Let the speed of A be x m.p.h.

" " " B " y m.p.h.

$2x + 2y = 26$ or $x + y = 13$

or $y = 13 - x$ ---- ①

also A walks one mile in $\frac{1}{x}$ hrs or in $\frac{60}{x}$ mints

B " " " " " $\frac{1}{y}$ hrs. " " $\frac{60}{y}$ mints

$\therefore \frac{60}{x} - \frac{60}{y} = 4\frac{1}{2}$ or $\frac{20}{x} - \frac{20}{y} = \frac{3}{2}$ or $40y - 40x = 3xy$ or

$40(y - x) = 3xy$ ---- ②

Substitute from ① in ②: $40(13 - 2x) = 3x(13 - x)$ or

$520 - 80x = 39x - 3x^2$ or $3x^2 - 119x + 520 = 0$ or

$(3x - 104)(x - 5) = 0$ or $x = 5$ and $x = \frac{104}{3} = 34\frac{2}{3}$

$\therefore y = 13 - x$ or $y = 13 - 5 = 8$ or $y = 13 - 34\frac{2}{3} = -21\frac{2}{3}$ inadmissible

$\therefore x = 5$ m.p.h.
$y = 8$ m.p.h. } Ans.

Shamash Secondary School
Mid-Year Examination, Feb.1967

Subject: Algebra
Class: 4th Year,Secondary

Date: 6/2/1967
Time: 8:30 - 11:30 a.m.

Attempt all questions:

1. Revolve into factors:

   (i) $3x^2-(4a+2b)x+a^2+2ab$ (6 marks)

   (ii) $8x^3-27y^3+z^3+18xyz$ (6 ")

   (iii) Divide $(a^2+b^2+c^2)(a+1)+(2ab-2ac)(a+1)-2abc-2bc$ by $(a+1)$ and express the quotient as a perfect square. (8 marks)

2. (i) If $x+y = 2a$ and $x-y=2b$, find in the shortest possible way, the value of $x^4+x^2y^2+y^4$. (10 marks)

   (ii) Find the value of p which will make the expression $2x^3+px^2-5x+2$ divisible by $(x+2)$ and find the other two factors.

   (10 marks)

3. (i) A man can row upstream at 'a' miles an hour and downstream at 'b' miles an hour. He rows up to a certain point and then returns to his starting point, and finds that his average speed is 's' miles an hour for the double journey. Express each of the letters in terms of the other two. Find the value of 'b' if a=2 and s=3.

   (10 marks)

   (ii) Solve the two simultaneous equations:

   $x^2+4y^2+80 = 15x+30y$ .....(1)

   $xy = 6$ . .....(2)

   (10 marks)

4. Two men started at the same time to meet each other from points which were 26 miles apart. If one took 4½ minutes longer than the other to walk a mile, and they met 2 hours after starting, find the speed of each in miles per hour.

   (20 marks)

(cont'd.p.2)...

Algebra. 4th Secondary. 6/2/1967

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5. (i) Draw on the same diagram the graphs of the function $4x-3$, and of the function $4x^2-4x-15$, taking ½ inch as one unit on the x-axis and one tenth of an inch as one unit on the y-axis. (8 marks)

(ii) From your diagram, find the roots of the two simultaneous equations $y = 4x-3$ .....(1)

$y = 4x^2-4x-15$ ....(2) (7 marks)

(iii) From the graph of the function $4x^2-4x-15$, find the roots of the equation $4x^2-4x-15=0$.

(5 marks)

27/12/1966

1. (a) $x = \frac{3ay-5bz}{3ay+5bz}$ $\therefore 3axy+5bxz = 3ay-5bz$ $\therefore 3ay-3axy = 5bxz+5bz$

$\therefore 3ay(1-x) = 5bz(x+1)$ $\therefore a = \frac{5bz}{3y} \cdot \frac{1+x}{1-x}$ Ans. 1

also $5bxz+5bz = 3ay-3axy$ $\therefore 5bz(x+1) = 3ay(1-x)$

$\therefore b = \frac{3ay}{5z} \cdot \frac{1-x}{1+x}$ Ans. 2 (8 marks)

(b) $\frac{a}{b} = k$ $\therefore \frac{4a-5b}{\sqrt{18a^2-4b^2}} = \frac{4\frac{a}{b}-5}{\frac{1}{b}\sqrt{18a^2-4b^2}} = \frac{4\frac{a}{b}-5}{\sqrt{18\frac{a^2}{b^2}-4}} = \frac{4k-5}{\sqrt{18k^2-4}}$ Ans.

(8 marks)

2. (a) $x = \frac{y}{y+1}$ and $y = \frac{a-2}{2}$. Prove that $x(y+2)+\frac{x}{y}+\frac{y}{x} = a$

$x = \frac{\frac{a-2}{2}}{\frac{a-2}{2}+1} = \frac{a-2}{a-2+2} = \frac{a-2}{a}$. Now substitute $x = \frac{a-2}{a}$, $y = \frac{a-2}{2}$

$\therefore$ the expression $x(y+2)+\frac{x}{y}+\frac{y}{x} = \frac{a-2}{a}\left(\frac{a-2}{2}+2\right)+\frac{\frac{a-2}{a}}{\frac{a-2}{2}}+\frac{\frac{a-2}{2}}{\frac{a-2}{a}}$

$\therefore$ the expression $= \frac{a-2}{a} \cdot \frac{a+2}{2} + \frac{2a-4}{a(a-2)} + \frac{a^2-2a}{2(a-2)} = \frac{a^2-4}{2a} + \frac{2(a-2)}{a(a-2)} + \frac{a(a-2)}{2(a-2)}$

$= \frac{a^2-4}{2a} + \frac{2}{a} + \frac{a}{2} = \frac{a^2-4+4+a^2}{2a} = \frac{2a^2}{2a} = a$ Q.E.D.

(8 marks)

(b) $\frac{1-1.4x}{0.2+x} = \frac{0.7(x-1)}{0.1-0.5x}$ multiply the first fraction (both numerator + Den.)

by 5 + the second fraction by 10, , we get

$\frac{5-7x}{1+5x} = \frac{7(x-1)}{1-5x}$ $\therefore (5-7x)(1-5x) = 7(1+5x)(x-1)$

$\therefore 5-32x+35x^2 = 7(5x^2-4x-1)$ or $35x^2-32x+5 = 35x^2-28x-7$

$\therefore 4x = 12$ $\therefore x = 3$ Ans.

(8 marks)

Solutions to 2nd Quarter Exam. in Algebra Cont(d)

27/12/1966

3 (a). $\dfrac{5}{6-\dfrac{5}{6-\dfrac{5}{6-x}}} = \dfrac{5}{6-\dfrac{5}{\dfrac{36-6x-5}{6-x}}} = \dfrac{5}{6-\dfrac{5(6-x)}{31-6x}}$

$\therefore \dfrac{5}{\dfrac{186-36x-30+5x}{31-6x}} = \dfrac{5(31-6x)}{156-31x} = \dfrac{155-30x}{156-31x}$

$\therefore \dfrac{155-30x}{156-31x} = x \quad \therefore 155-30x = 156x-31x^2$

$\therefore 31x^2-186x+155=0 \quad \therefore x^2-6x+5=0$

$\therefore (x-1)(x-5)=0 \quad \therefore x=1 \text{ or } x=5$ Ans. (8 marks)

(b) $3x^3+x^2+4=8x \quad \therefore 3x^3+x^2-8x+4=0$

when $x=1$, then LHS $= 3+1-8+4=0 \quad \therefore (x-1)$ is a factor

$\therefore 3x^3+x^2-8x+4 = 3x^2(x-1)+4x(x-1)-4(x-1)$

$\therefore 3x^3+x^2-8x+4 = (x-1)(3x^2+4x-4) = (x-1)(3x-2)(x+2)=0$

$\therefore x=1, \; x=-2, \; x=\frac{2}{3}$ Ans. (10 marks)

(c) $x^2y^2+192 = 28xy \quad \ldots\ldots (1)$

$x+y = 8 \quad \ldots\ldots (2)$

$x^2y^2-28xy+192=0 \quad \therefore (xy-16)(xy-12)=0$

$\therefore xy=16 \text{ or } xy=12.$

squaring eq. (2), $x^2+2xy+y^2=64 \quad \ldots (3)$

$-4xy = -48$ | or $-4xy = -64$

$\therefore x^2-2xy+y^2=16$ || or $x^2-2xy+y^2=0$

$(x-y)^2=16$ || $(x-y)^2=0$

$x-y=\pm 4$ || $x-y=0$

Now $x+y=8$, $x-y=4$ || $x+y=8$, $x-y=-4$ || $x+y=8$, $x-y=0$ ||

$2x=12$, $x=6$, $y=2$ } Ans.1 || $2x=4$, $x=2$, $y=6$ } Ans.2 || $2x=8$, $x=4$, $y=4$ } Ans.3

(10 marks)

4. $\frac{y}{x}$ hrs. = No. of hrs. taken by fast train to cover the distance y mls.

Let v m.p.h. be the speed of the slower train.

$\therefore \frac{z}{v}$ hrs. = No. of hrs taken by slower train to travel distance z mls.

$\therefore \frac{z}{v} = \frac{y}{x} + t \quad \therefore xz = vy + txv \quad \therefore v(y+tx) = xz$

$\therefore v = \frac{xz}{y+tx}$ = the difference between speeds $= (x-v)$ m.p.h.

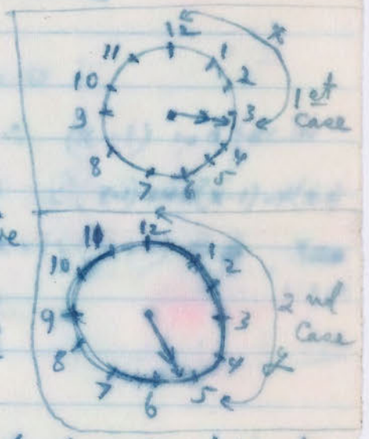
$\therefore$ diff. between the two speeds $= x - \frac{xz}{y+tx} = \frac{xy+tx^2-xz}{tx+y} = \frac{tx^2+x(y-z)}{tx+y}$

(20 Marks) Ans.

5. Let the time when he started be x minutes after 3

$\therefore x = 15 + \frac{x}{12} \quad \therefore 12x = 180 + x$

$\therefore 11x = 180 \quad \therefore x = \frac{180}{11} = 16\frac{4}{11}$ minutes



Let the time when he finished $= y$ minutes after five

$\therefore y = 25 + \frac{y}{12} \quad \therefore 12y = 300 + y$

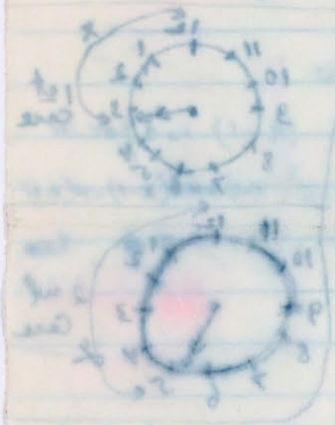
$\therefore 11y = 300 \quad \therefore y = \frac{300}{11} = 27\frac{3}{11}$ minutes after five

$\therefore$ he began at $16\frac{4}{11}$ min. after 3, and ended at $27\frac{3}{11}$ min past five.

He walked for a period of 2 hrs $10\frac{10}{11}$ minutes Ans.

$= (\frac{5}{2}$ hrs. $27\frac{3}{11}$ min $-$ 3 hrs. $16\frac{4}{11}$ min$)$ ←

(20 marks)



Shamash Secondary School

2nd Quarter Examination, December, 1966

Subject: Algebra
Class: 4th Scientific Year

Date: 27/12/1966
Time: 11:00 - 12:30 morning.

All questions are to be attempted.

1. (a) Given that $x = \frac{3ay - 5bz}{3ay + 5bz}$; make a, b respectively the subject of the formula. (8 marks)

   (b) If $\frac{a}{b} = k$, express $\frac{4a - 5b}{\sqrt{18a^2 - 4b^2}}$ in terms of k. (8 marks)

2. (a) Prove that $x(y+2) + \frac{x}{y} + \frac{y}{x}$ is equal to a, if $x = \frac{y}{y+1}$ and $y = \frac{a-2}{2}$ (8 marks)

   (b) Solve the equation $\frac{1-1.4x}{0.2+x} = \frac{0.7(x-1)}{0.1-0.5x}$ (8 marks)

3. (a) Solve $\dfrac{5}{6-\dfrac{5}{6-\dfrac{5}{6-x}}} = x$ (8 marks)

   (b) Solve $3x^3 + x^2 + 4 = 8x$ (10marks)

   (c) Solve $x^2y^2 + 192 = 28xy$ .....(1) (10 marks)

   $x + y = 8$ .....(2)

4. A fast train travelling at x miles an hour takes t hours less to travel y miles than a slower one takes to travel z miles.
   Find the difference between their speeds in terms of x, t, y and z.

   (20 marks)

5. A man started for a walk when the hands of his watch were coincident between three and four o'clock. When he finished, the hands were again coincidents between five and six o'clock. What was the time when he started, and how long did he walk ?

   (20 marks)

Shamash Secondary School

1st Quarter Examination, November, 1966.

Subject: Algebra — Date: 8/11/1966

Class: 4th Secondary, Scientific Section. — Time: 12:00 - 1:30 p.m.

All questions are to be attempted

1. (i) Given $\frac{\sqrt{xy}+8x}{\sqrt{2x}-y\delta} = \frac{5}{4}$ Solve each x, y, δ in terms of the other two (6 marks)

(ii) Simplify by removing brackets

$$35\left[\frac{3x-4y}{5} - \frac{1}{10}\left\{3x - \frac{2}{7}(7x-4y)\right\}\right] + 8(y-2x)$$ (8 marks)

2. (i) Resolve into two factors (if possible) each of the following expressions:

① $a^7 - b^7$ ② $a^5 + b^5$ ③ $a^6 - b^6$ ④ $a^4 + b^4$ (12 marks)

(ii) Write down by inspection the quotient of $\frac{(2a)^2 - (3b)^2}{2a - 3b}$ (4 marks)

3. (i) Solve the equation $2.4 = \frac{0.24}{0.6} - \frac{0.16x - 7.6}{0.8}$ (9 marks)

(ii) Walking $5\frac{1}{2}$ miles an hour, I start $2\frac{1}{2}$ hours after a friend whose pace is $3\frac{1}{2}$ miles an hour. How long shall I be in overtaking him? (8 marks)

4. (i) How many days will "x" men take to mow "x" acres if "b" boys can mow "y" acres in "d" days and each man's work equals that of "n" boys? (9 marks)

(ii) Find the square root of:

$$16x^4 + \frac{16}{3}x^2y + 8x^2 + \frac{4}{9}y^2 + \frac{4}{3}y + 1$$

showing your steps neatly. (8 marks)

5. (i) Divide $3x^3 - 2x^2 + Bx - 26$ by $x - 2$.
Hence find the value of 'B' that makes the expression: $3x^3 - 2x^2 + Bx - 26$ factorable into $(x-2)$, and find the other factor.

(9 marks)

(ii) A man can swim at x m.p.h. in still water. His ~~speed~~ rate increases y m.p.h. when he swims with the current, and decreases y m.p.h. when swims against the current. The difference in his time to swim 2 miles with the current and 2 miles against the current is z hours. Find a formula for z in terms of x and y.

(7 marks)

6. Give the English equivalent to the following:

١. أعداد زوجية وأعداد فردية
٢. الجزء الصحيح من عدد كسري
٣. مقلوب العدد
٤. معامل حرفي
٥. أساس القوة وأس القوة
٦. الخط المستقيم، الخط المنحني
٧. نقل الحدود من طرف إلى الطرف الآخر
٨. إن قيمة المقدار ٦٤١٩ صحيحة لتقريب ثلاثة أرقام معنوية هي
٩. محيط المثلث
١٠. وتر القوس في دائرة

(20 marks)

$\frac{xy+yz+zx}{\sqrt{2x-yz}} = \frac{5}{4}$

$x(16y+16z-50) = -25yz$

$x = \frac{-25yz}{16y+16z-50}$ Ans. I (2 marks)

$y(16x+25z) = 50xz - 16xyz$ $\therefore y = \frac{50xz-16xyz}{16x+25z}$ Ans. II (2 marks)

$z(16x+25y) = 50xy - 16xyz$ $\therefore z = \frac{50xy-16xyz}{16x+25y}$ Ans. III (2 marks)

(ii) simplify by removing brackets

$= 35\left[\frac{3x-4y}{5} - \frac{1}{10}\left\{2x - 5x + \frac{20}{7}y\right\}\right] + 8y - 16x$

$= 7(3x-4y) - \frac{2}{5}\left(2x - 5x + \frac{20}{7}y\right) + 8y - 16x$

$= 21x - 28y - \frac{2}{5}x + 4x - 8y + 8y - 16x$

$= 12x - 28y$ Ans. (8 marks)

2 (i) ① $a^4 - b^4 = (a-b)(a^3 + a^2b + ab^2 + b^3)$ Ans. I 3

② $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ Ans. II

③ $a^5 - b^5 = (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$ Ans. III

④ $a^4 + b^4$ = not factorable

(ii) $\frac{(2a)^5 - (3b)^5}{2a-3b} = (2a)^4 + (2a)^3(3b) + (2a)^2(3b)^2 + (2a)(3b)^3 + (3b)^4$ 4

$= 16a^4 + 24a^3b + 36a^2b^2 + 54ab^3 + 81b^4$

3 (i) $2.4 = \frac{0.24}{0.6} - \frac{0.1(x-7.6)}{2.5}$ $\therefore 2.4 = \frac{24}{60} - \frac{(x-7.6)}{0.5}$

$\frac{24}{5} = \frac{2}{5} - \frac{(x-7.6)}{5}$ $\therefore 2.4 = 0.4 - \frac{(x-7.6)}{5}$

$= -\frac{(5x-38)}{24}$ $\therefore 48 = -5x + 38$ $\therefore 5x = 180$

$\therefore x = 36$ Ans. (8 marks)

$(2\frac{1}{2})(3\frac{1}{2})$

$AB = \frac{5}{2} \times \frac{7}{2} = \frac{35}{4}$ miles

Let t hours = time taken in A to B to reach C

$\therefore 5\frac{1}{2}t - 3\frac{1}{2}t = \frac{35}{4} \therefore t(5\frac{1}{2} - 3\frac{1}{2}) = \frac{35}{4} \therefore 2t = \frac{35}{4} \therefore t = \frac{35}{8}$ hrs.

$\therefore t = 4\frac{3}{8}$ hours or 4 hours 22.5 minuts. Ans. (8 marks)

4. (i)

boys	men	acres	days
b		y	d
	m	a	?

1 man = u boys

(9 marks)

b boys = $\frac{b}{u}$ men

men	acres	days
$\frac{b}{u}$	y	d
m	a	?
g	D	

$\therefore$ No. of days = $\frac{d \cdot a}{y} \cdot \frac{\frac{b}{u}}{m}$

$= \frac{adb}{myu}$ days Ans.

(ii)

$16x^4$ | $8x^2 + \frac{16}{3}x^2y + \frac{4}{3}y + \frac{4}{9}y^2 + 1$ | $4x^2 + y + \frac{2}{3}y$

$8x^2$

$8x^2 + 1$ | $\frac{16}{3}x^2y + \frac{4}{3}y + \frac{4}{9}y^2 + 1$

$\frac{16}{3}x^2y$

$8x^2 + 2 + \frac{2}{3}y$ | $\frac{4}{3}y + \frac{4}{9}y^2$

$\frac{4}{3}y$

$\frac{16}{3}x^2y + \frac{4}{3}y + \frac{4}{9}y^2$

(8 marks)

5 (i)

$x-2$ | $3x^3 - 2x^2 + Bx - 26$ | $3x^2 + 4x + 8 + \frac{B}{x-2}$

$3x^3 - 6x^2$

$4x^2 + Bx - 26$

$4x^2 - 8x$

$(8+B)x - 26$

$(8+B)x - 16 - 2B$

$2B - 10$

Hence $2B - 10 = 0 \therefore B = 5$

Hence the other factor is the quotient $3x^2 + 4x + 8 + 5$ or

$3x^2 + 4x + 8 + 5$ or

$3x^2 + 4x + 13$ Ans. (9 marks)

(ii) $\frac{z}{x-y} - \frac{z}{x+y} = z$ or $z = \frac{2x + 2y - 2x + 2y}{(x-y)(x+y)}$ or $z = \frac{4y}{x^2 - y^2}$ Ans. (8 marks)

Diama d Secondary School

2nd Quarter Examination, December, 1966

Subject: Algebra Date: 27/12/1966

Class: 4th Scientific ... Time: 11:00 - 12:30 morning

All questions are to be attempted.

1. (a) Given that $x = \frac{3ay - 5b^2}{3y + 5b}$; make a, b respectively the subject of the formula. (9 marks)

(b) If $\frac{a}{b} = k$, express $\frac{4a - 5b}{\sqrt{15a^2 - 4b^2}}$ in terms of k. (9 marks)

2. (a) Prove that $x(y+z) + \frac{x}{y} + \frac{y}{x}$ is equal to z, if

$x = \frac{y}{y+1}$ and $y = \frac{z-2}{2}$ (8 marks)

(b) Solve the equation $\frac{1 - 2.4x}{0.2 + x} = \frac{0.7(x-1)}{0.1 - 0.5x}$ (9 marks)

3. (a) Solve $\frac{5}{6 - \frac{5}{6-x}} = x$ (9 marks)

(b) Solve $3x^3 + x^2 + 4 = 3x$ (10 marks)

(c) solve $x^2y^2 + 192 = 28xy$ ..... (i)

$x + y = 8$ ..... (ii) (10

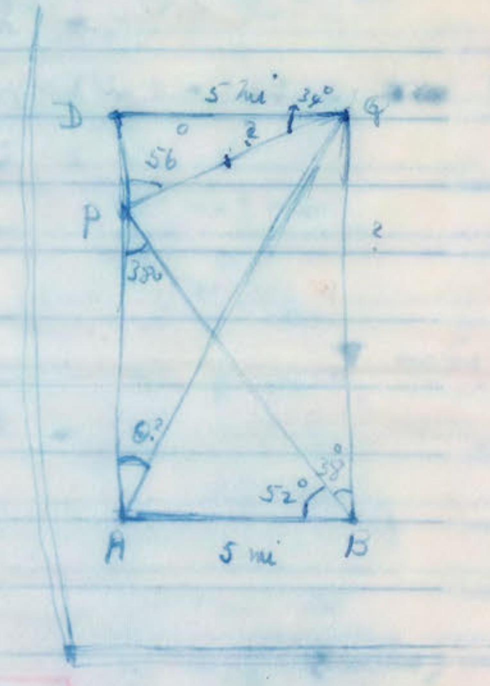
4. A fast train travelling at x miles an hour takes t hours less to travel y miles than a slower one takes to travel z miles. Find the difference between their speeds in terms of (20 marks)
x, t, y and z.

5. A man started for a walk when the hands of his watch were coincident between ... and four o'clock. When he finished, the hands were ... coincident between five and six o'clock. What was the time when he started, and how long did he walk?

(20 marks)

Solution of Trigonometry Questions
4th year Conditional Exam Sept., 1966

Q.4. $PQ = ?$, $\angle\theta = ?$



In $\triangle PDQ$, $\frac{PQ}{5} = \operatorname{cosec} 56^\circ$

$\therefore PQ = 5 \operatorname{cosec} 56^\circ$

$\frac{DP}{5} = \tan 34^\circ$ $\therefore DP = 5 \tan 34^\circ$

$\frac{AP}{5} = \tan 52^\circ$ $\therefore AP = 5 \tan 52^\circ$

$AD = BQ = DP + AP = 5 \tan 34^\circ + 5 \tan 52^\circ = 5(\tan 34^\circ + \tan 52^\circ)$

from $\triangle AQD$, $\tan\theta = \frac{5}{AD}$ $\therefore \tan\theta = \frac{5}{5(\tan 34^\circ + \tan 52^\circ)}$

$\therefore \tan\theta = \frac{1}{\tan 34^\circ + \tan 52^\circ} =$

$PQ = 5 \operatorname{cosec} 56^\circ = 5 \times 1.2062 =$ 6.0310 miles Ans. 1

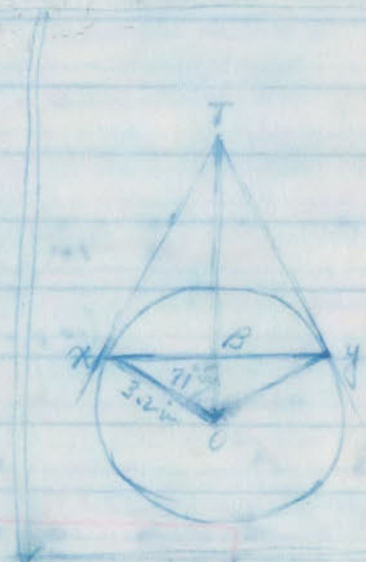
$BQ = 5(\tan 34^\circ + \tan 52^\circ) = 5(0.6745 + 1.2799) = 5 \times 1.9544 =$ 9.7720 miles Ans. 2

$\tan\theta = \frac{1}{\tan 34^\circ + \tan 52^\circ} = \frac{1}{0.6745 + 1.2799} = \frac{1}{1.9544} = 0.51165 = 0.5117$ Correct to 4 dec. pl.

$\therefore \theta = 27^\circ 6'$ Ans. 3

Q.5. $\angle XOY = 143^\circ 28'$

$\therefore \angle XOT = \frac{143^\circ 28'}{2} = 71^\circ 44'$



In $\triangle OXT$

$\frac{3.2}{OT} = \cos 71^\circ 44'$

$\therefore OT = \frac{3.2}{\cos 71^\circ 44'} = 3.2 \sec 71^\circ 44'$

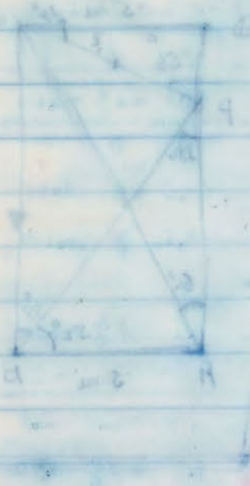
$\therefore OT = 3.2 \times 3.1903 = 10.20896$ in $=$ 10.21 in Correct to 2 dec. pl. Ans. 1

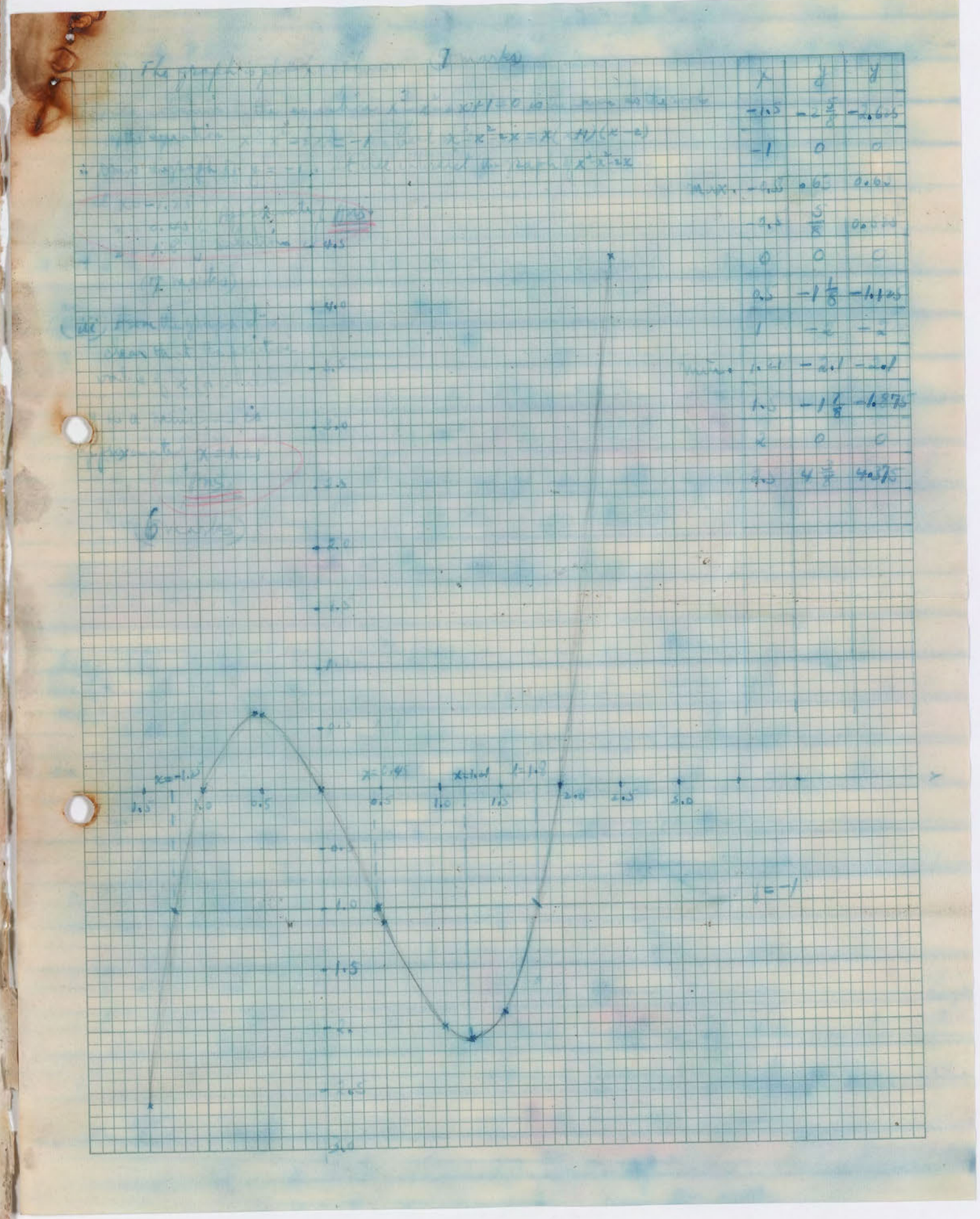
in $\triangle OXB$, $\frac{XB}{3.2} = \sin 71^\circ 44'$ $\therefore XB = 3.2 \sin 71^\circ 44'$

$\therefore XY = 2 XB = 2 \times 3.2 \sin 71^\circ 44' = 6.4 \sin 71^\circ 44' = 6.4 \times 0.9496$

$= 6.07744$ in $= 6.08$ in Correct to 2 dec.

Ans. 2




x = -1
x = 1

[illegible], 1960

1. (i) $x^{16}-y^{16} = (x^8-y^8)(x^8+y^8) = (x^4-y^4)(x^4+y^4)(x^8+y^8) = (x^2-y^2)(x^2+y^2)(x^4+y^4)(x^8+y^8)$

$= (x-y)(x+y)(x^2+y^2)(x^4+y^4)(x^8+y^8)$ Ans. (8 marks)

(ii) $\dfrac{(x^2+y^2+z^2)(x+1)+(2xy-2xz)(x+1)-2xyz-2yz}{x+1}$

$= \dfrac{(x^2+y^2+z^2)(x+1)+(2xy-2xz)(x+1)-2yz(x+1)}{(x+1)} = x^2+y^2+z^2+2xy-2xz-2yz$

$= (x+y-z)^2$ Ans. (8 marks)

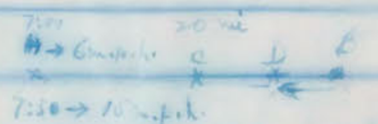
(iii) $a = \dfrac{x^2-2x+2}{x(2-x)}$ and $b = \dfrac{x-1}{x(2-x)}$

$a^2-4b^2 = (a+2b)(a-2b) = \left(\dfrac{x^2-2x+2}{x(2-x)}+\dfrac{2x-2}{x(2-x)}\right)\left(\dfrac{x^2-2x+2}{x(2-x)}-\dfrac{2x-2}{x(2-x)}\right)$

$= \left(\dfrac{x^2-2x+2+2x-2}{x(2-x)}\right)\left(\dfrac{x^2-2x+2-2x+2}{x(2-x)}\right) = \left(\dfrac{x^2}{x(2-x)}\right)\left(\dfrac{x^2-4x+4}{x(2-x)}\right) = \dfrac{x^2(x-2)^2}{x^2(2-x)^2}$

$= \dfrac{x^2(2-x)^2}{x^2(2-x)^2} = 1$ Ans. (8 marks)

2(a) (i) dist. from A of man at 7:40 =



$\frac{40}{60} \times 6 = 4$ miles Ans. 1 (1 mark)

distance of son at 7:40 $= \frac{10}{60} \times 15 = 2\frac{1}{2}$ miles Ans. (1 mark)

(ii) distance of man from A, t hours after 7:00 $= 6t$ miles Ans. 1 (1 mark)

" " son " " , t " " 7:00 $= 15(t-\frac{1}{6})$ miles Ans. [illegible]

Let the time be t hrs. after 7:00 a.m. when the son overtakes the father at the C

$\therefore 6t = 15(t-\frac{1}{6})$ or $6t = 15t - \frac{15}{6}$ or $9t = \frac{15}{6}$ $\therefore t = \frac{15}{18}$ hrs. $= \frac{5}{6}$ [illegible]

$\therefore$ the time at the instant of overtaking is $7\frac{5}{6}$ h or 7:50 a.m. Ans.

(5 marks)

(b) the boy takes $\frac{20}{15}$ hrs. to cover the distance AB or $\frac{4}{3}$ hrs. Now let the length of time taken by the boy to meet his father again on his way back from B be T hours

$\therefore$ [illegible] distance covered by the boy from B to the meeting point D is $= 15T$ $\therefore$ distance covered by the father upto till this instant $= (20-15T)$ miles

$\therefore$ the Time taken by the father to cover distance AD $= \frac{20-15T}{6}$ hrs.

$\therefore \frac{20-15T}{6} =$ the no. of hours which have elapsed after 7:00 a.m.

$\therefore \frac{20-15T}{6} = \frac{1}{2} + \frac{4}{3} + T + \frac{1}{2}$ or $20-15T = 6+8+6T$ $\therefore 21T = 6$ $\therefore T = \frac{6}{21}$ hrs $= \frac{2}{7}$

$\therefore T = \frac{2}{7}$ hrs $\therefore$ The time at this instant $= (\frac{1}{2}+\frac{4}{3}+\frac{2}{7})$ hrs after 7:00 a.m. [illegible] $\frac{21+56+12}{42}$ [illegible]

[illegible]

or $T = \frac{2}{7}$ hrs. ∴ the time at this instant is $= \left(1+\frac{4}{5}+\frac{2}{7}\right)$ hrs after 7:00 a.m. $= \left(1+\frac{4}{3}+\frac{2}{7}\right)$

$= \frac{21+28+6}{21} = \frac{55}{21} = 2\frac{13}{21}$ hrs after 7:00 a.m.

∴ the time at the instant is $9:37\frac{1}{7}$ a.m. $= 9:37$ a.m. to the nearest minute

(10 marks)

2 (i) $\log \sin 17°14' = \bar{1}.4717$
$\log \cos 0°11' = \bar{1}.9944$
$\log 1.003 = 0.0012$
$\log 15.04 = 1.1772$

$2\log \sin 17°14' = \bar{2}.9434$
$3\log \cos 0°11' = \bar{1}.9832$
$\log Num = \bar{2}.9266$
$\log Den = 5.8896$
$8\log x = \bar{7}.0370$
$\log x = \bar{1}.129625 = \bar{1}.1296$ Correct to 4 dec. pl.
$\therefore x = 0.1348$ Ans. (10 marks)

$3\log 1.003 = 0.0036$
$\log 15.04 = 1.1772$
$+\quad 5.8860$
$\log Den = 5.8896$

(ii) $(\log x)^2 - 4\log x + 4 = 0$ $\therefore (\log x - 2)^2 = 0$ $\therefore \log x = 2$ $\therefore x = 100$ Ans.

4(i) Between 99 and 4820 the first number which is divisible by 13 is $(\frac{99}{13} = 7+\frac{8}{13})$ $8\times13 = 104$ (since $\frac{92}{13} = 7\frac{1}{13}$). The last number divisible by 13 is 4810 (since $\frac{4815}{13} = 370\frac{5}{13}$). Hence the numbers required are $104, 117, 130, \ldots, 4810$

$l = a + (n-1)d$ $\therefore 4810 = 104 + (n-1)13$ $\therefore 13n = 4810 - 104 + 13$

or $13n = 4719$ $\therefore n = \frac{4719}{13} = 363$ = no. of terms

$\therefore S = \frac{n}{2}(a+l) = \frac{363}{2}(104+4810) = \frac{363}{2} \times 4914 = 363 \times 2457$

$= 891891$ Ans. (10 marks)

(ii) first × sixth = second × fourth $\therefore a \cdot ar^5 = 99 \cdot ar^3$ $\therefore ar^2 = 99$ ①

$a + ar^2 = -286$ or $a(1+r^2) = -286$ ②

Dividing: $\frac{ar^2}{a(1+r^2)} = \frac{99}{-286}$ $\therefore \frac{r^2}{1+r^3} = -\frac{9}{26}$ $\therefore 9+9r^3 = -9r^2$

or $9r^3 + 26r^2 + 9 = 0$ By trial & error $r = -3$ satisfies the equation

$\therefore r+3$ is a factor of L.H.S. $\therefore (r+3)(9r^2 - r + 3) = 0$ the second factor is no (

$\therefore r = -3$. From ① $ar^2 = 99$ $\therefore 9a = 99$ $\therefore a = 11$ Ans.)

P.T.O

$$S_n = \frac{a(r^n-1)}{r-1} = \frac{11[(-3)^7-1]}{-3-1} = \frac{11[1-(-3)^7]}{1+3} = \frac{11[1-(-2187)]}{4} = \frac{11(1+2187)}{4}$$

$$= \frac{11 \times 2188}{4} = 11 \times 547 = 6017 \quad \text{Ans.}$$

(10 marks)

Shamagh Secondary School
Conditional Examination, Sept. 1966

Subject: Algebra
Class : 4th Year Secondary

Date: 8/9/1966
Time: 8:00 – 11:00 a.m.

Attempt all questions:-

1. (i) Resolve $x^{16}-y^{16}$ into five factors. (8 marks) 6

(ii) Divide $(x^2+y^2+z^2)(x+1)+(2xy-2xz)(x+1)-2xyz-2yz$ by $x+1$ and express the quotient as a perfect square. (8 marks) 6

(iii) If $a = \frac{x^2-2x+2}{x(2-x)}$ and $b = \frac{x-1}{x(2-x)}$, find the numerical value of $a^2 - 4b^2$ (9 marks). 8

2. (a) A man sets out at 7 a.m. from a town A to drive his horse and cart to B, a distance of 20 miles. His average speed is 6 m.p.h. at 7:30 a.m. his son leaves A by bicycle on the same road, riding at an average speed of 15 m.p.h. Write down the distance from A of each (i) at 7:40 a.m., (ii) t hours after 7 a.m. Calculate the time when the boy overtakes his father. (10 marks)

(b) When the boy reaches B, he spends half an hour there and then rides back along the same road at the same average speed. Calculate the time when he meets his father. (10 marks)

3. (i) Compute by logarithms the value of the following:-

$$x = \sqrt[8]{\frac{(\text{Sin } 17^\circ 14')^2 (\text{Cos } 9^\circ 11')^3}{(1.003)^3 (15.04)^5}}$$ (10 marks)

(ii) Find the value of x from the following equation:

$(\log x)^2 - 4 \log x + 4 = 0$ (10 marks)

4. (i) Find the sum of all the numbers between 92 and 4815 which are exactly divisible by 13. How many of these numbers are there? (10 marks).

(ii) The product of the first and sixth terms of a geometric progression is equal to 99 times the fourth term and the sum of the first and fourth terms is (-286). Find the first term and the sum of the first seven terms. (10 marks)

5. (i) Draw the graph of $y=x(x+1)(x-2)$ between $x=-1.5$ and $x=+2.5$, taking one inch as unit for x and one inch as unit for y. (9 marks) 7

From your graph obtain:-

(ii) Approximate solutions for the equation $x^3-x^2-2x+1=0$ (8 marks) 7

(iii) The positive value of x for which the expression $x(x+1)(x-2)$ is a minimum. (8 marks). 6

6

[illegible] $my^4 + 16mx^4 - 12mxy^2 = m(16x^4 - 12x^2y^2 + y^4) = m(16x^4 - 8x^2y^2 + y^4 - 4x^2y^2) =$
$= m[(4x^2-y^2)^2 - (2xy)^2] = m(4x^2-y^2+2xy)(4x^2-y^2-2xy) = m(4x^2+2xy-y^2)(4x^2-2xy-y^2)$ Ans. 1

(ii) $a^2b^2x^2 - a^2b^2 - 2abx^2 + 2ab + x^2 - 1 = a^2b^2(x^2-1) - 2ab(x^2-1) + (x^2-1)$
$= (x^2-1)(a^2b^2 - 2ab + 1) = (x-1)(x+1)(ab-1)^2$ Ans. 2

(iii) $27x^6y^9 + 64y^3 = y^3(27x^6y^6 + 64) = y^3[(3x^2y^2)^3 + 4^3]$
$= y^3(3x^2y^2+4)(9x^4y^4 - 12x^2y^2 + 16) = y^3(3x^2y^2+4)(9x^4y^4 + 24x^2y^2 + 16 - 36x^2y^2)$
$= y^3(3x^2y^2+4)[(3x^2y^2+4)^2 - (6xy)^2] = y^3(3x^2y^2+4)(3x^2y^2+6xy+4)(3x^2y^2-6xy+4)$
Ans. 3

(b) $2x^3 + px^2 + qx + 1$ will be divisible by $(x-1)$ if the expression $= 0$ when $x = 1$
$\therefore 2 + p + q + 1 = 0$ ... (1) also when $(x+1) = 0$ then $x = -1$ when the expression
$\therefore -2 + p - q + 1 = 0$ ... (2)
$\therefore 2p + 2 = 0 \quad \therefore p + 1 = 0 \quad \therefore p = -1$ from equation (1) $2 + p + q + 1 = 0$ | $p = -1$
$\therefore 2 - 1 + q + 1 = 0 \quad \therefore q = -2$ $\therefore$ the expression is $2x^3 - x^2 - 2x + 1 = 0$ | $q = -2$
$\therefore (2x^3 - 2x) - (x^2 - 1) = 0$ or $2x(x^2-1) - (x^2-1) = 0 \quad \therefore (x^2-1)(2x-1) = 0$
$\therefore (x-1)(x+1)(2x-1) = 0$ $\therefore$ the third factor is $(2x-1)$ Ans. 3b
Another method would be to divide $(2x^3 + px^2 + qx + 1)$ actually by $(x-1)$ and [illegible] getting a remainder of $(2+p+q+1)$ in the first case and a remainder of $(-2+p-q+1)$ [illegible] second case. Equating each remainder to zero, we obtain the same answer.

2. (a) $ax^2 + bx + c = 0 \quad \therefore a(x^2 + \frac{b}{a}x + \frac{c}{a}) = 0 \quad \therefore a\left[x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} + \frac{c}{a} - \frac{b^2}{4a^2}\right] = 0$
$\therefore a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2-4ac}{4a^2}\right] = 0 \quad \therefore a\left[\left(x+\frac{b}{2a}\right)^2 - \left(\frac{\sqrt{b^2-4ac}}{2a}\right)^2\right] = 0$
$\therefore a\left[\left(x + \frac{b}{2a} + \frac{\sqrt{b^2-4ac}}{2a}\right)\left(x + \frac{b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}\right)\right] = 0$
$\therefore x + \frac{b + \sqrt{b^2-4ac}}{2a} = 0 \quad \therefore x_1 = -\frac{b + \sqrt{b^2-4ac}}{2a}$
or $x + \frac{b}{2a} - \frac{\sqrt{b^2-4ac}}{2a} = 0 \quad \therefore x_2 = \frac{-b + \sqrt{b^2-4ac}}{2a}$
$\therefore$ the sum of the two roots $= x_1 + x_2 = \frac{-b - \sqrt{b^2-4ac}}{2a} + \frac{-b + \sqrt{b^2-4ac}}{2a} = \frac{-2b}{2a} = -\frac{b}{a}$ Ans. 2
the product [illegible] $= x_1 x_2 = \left(\frac{-b - \sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b + \sqrt{b^2-4ac}}{2a}\right)$
$= \frac{(-b)^2 - (b^2-4ac)}{4a^2} = \frac{b^2 - b^2 + 4ac}{4a^2} = \frac{4ac}{4a^2} = \left(\frac{c}{a}\right)$ Ans. 2

2. (b) $3\times10^{2x} - 13\times10^{x} + 4 = 0$ let $10^x = y$ then $3y^2 - 13y + 4 = 0$

$\therefore (3y-1)(y-4) = 0 \therefore y = 4$ or $y = \frac{1}{3}$

$\therefore 10^x = 4$ hence $x\log 10 = \log 4 \therefore x = \log 4 = 0.6021$ Ans.1.

$10^x = \frac{1}{3} \therefore x\log 10 = \log 1 - \log 3$ or $x = -\log 3 \therefore x = -0.4771$ Ans.2

3 (i) $\log_3(2x^2 - 7x) = 2 \therefore 2x^2 - 7x = 3^2$ or $2x^2 - 7x - 9 = 0$

$\therefore (2x-9)(x+1) = 0 \therefore x = \frac{9}{2} = 4.5$ Ans.1

or $x = -1$ Ans.2

(ii) $(\log_2 9)(\log_3 32) = ?$ Let $\log_2 9 = x \therefore 9 = 2^x$ or $3^2 = 2^x$

$\therefore x\log 2 = 2\log 3 \therefore x = \frac{2\log 3}{\log 2}$

also let $\log_3 32 = y \therefore 32 = 3^y$ or $2^5 = 3^{2y} \therefore 2y\log 3 = 5\log 2$

$\therefore y = \frac{5\log 2}{2\log 3}$

Now $(\log_2 9)(\log_3 32) = x \cdot y = \frac{2\log 3}{\log 2} \cdot \frac{5\log 2}{2\log 3} = 5$ Ans.

(iii) $y = \sqrt[7]{\dfrac{(\tan 19°45')^2 \times (\cos 77°16)^3}{(3.004)^5 \times (50.06)^3}}$

a more direct method would be as follows:

$(\log_3 9)(\log_3 32) = \frac{\log 9}{\log 2} \cdot \frac{\log 32}{\log 9} = \frac{5\log 2}{\log 2}$ = 5 Ans.

$\log \tan 19°45' = \bar{1}.5551$

$\log \cos 77°16 = \bar{1}.3432$

$\log 3.004 = 0.4777$

$\log 50.06 = 1.6995$

$2\log \tan 19°45' = \bar{1}.1102$

$3\log \cos 77°16 = \bar{2}.0296$

$\log \text{Num} = \bar{3}.1398$

$\log \text{Den} = 7.4870$

$7\log y = \bar{1}.6528$

$\log y = \bar{2}.52183$

$y = 0.03325$

$= 3.325 \times 10^{-2}$ Ans.

$5\log 3.004 = 2.3885$

$3\log 50.06 = 5.0985$

$\log \text{Den} = 7.4870$

$y = 0.03437$

$= 3.437 \times 10^{-2}$ Ans.

$2\log \tan 19°45' = \bar{1}.1102$

$3\log \cos 77°16 = \bar{2}.0296$

$\log \text{Num} = \bar{3}.1398$

$\log \text{Den} = 7.4870$

$7\log y = \bar{1}.6528$

$\log y = \bar{2}.52183$

$y = 0.03325$

$= 3.325 \times 10^{-2}$ Ans.

Let the 1st term be $= a$ & the Common difference be $= d$

$T_6 = a+7d$ $\left.\right\}$ $\therefore a+7d = 6(a+2d)$ $\therefore a+7d = 6a+12d$

$T_3 = a+2d$ $\left.\right\}$ $\therefore 5d = -5a$ $\therefore d = -a$

$\therefore$ Second term $= a+d = a-a = 0$ Ans.

(ii) ~~the 1st day the invalid walks 18 inches~~

the 1st day the invalid walks 18 inches } and so on.
the 2nd " " " " 36 inches }

$\therefore$ we have a Geometric progression in which

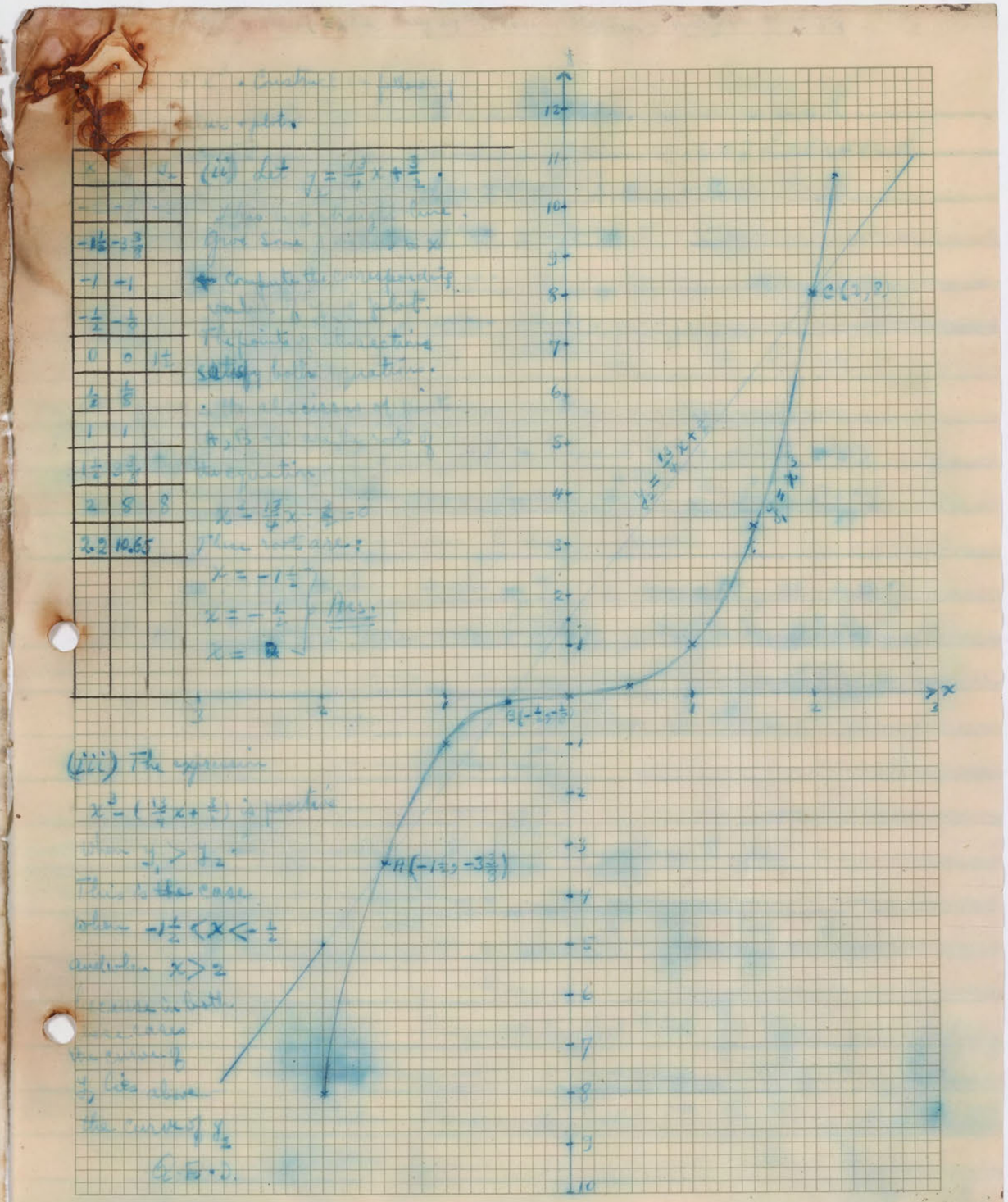
$a = 18$ inches, $l_n = 512$ yards $= 512 \times 36$ inches, and $r = 2$

Since $l_n = ar^{n-1}$ $\therefore 512\times36 = 18(2)^{n-1}$ or $2^{n-1} = \frac{512\times36}{18}$ or

$2^{n-1} = 512\times2 = 2^{10}$ $\therefore 2^{n-1} = 2^{10}$ $\therefore n-1 = 10$ or $n = 11$

$\therefore$ the invalid can take a walk of 512 yards on the 11th day (after 10 days)

Ans.


x
y
H(-1½, -3⅜)

Nour Basri 45

Shamash Secondary School
Final Examination, May, 1966.

Subject: Algebra
Class : 4th Year, Scientific.

Date: 29/5/1966
Time: 8:00 - 11:00 a.m.

Answer all f i v e questions:

1. (a) Factor the following:

(i) $my^4 + 16mx^4 - 12mx^2y^2$ (4 marks)

(ii) $a^2b^2x^2 - a^2b^2 - 2abx^2 + 2ab + x^2 - 1$ (4 marks)

(iii) $27x^6y^9 + 64y^3$ (4 marks)

(b) Find the value of p and q which will make the expression $2x^3 + px^2 + qx + 1$ divisible by (x-1) and (x+1), and find the third factor. (8 marks)

2. (a) Use the method of completing the square to show that the sum of the roots of the equation $ax^2+bx+c=o$ is equal to $(-\frac{b}{a})$ and that their product is equal to $(\frac{c}{a})$. (10 marks)

(b) Find the value of x from the following equation:

$3.10^{2x} - 13.10^{x} + 4 = o$ (10 marks)

3. Solve only <u>two</u> sections from the following <u>three</u> sections:

(i) Find the value of x from the following equation:

$\log_3 (2x^2-7x) = 2$ (10 marks)

(ii) Without using tables evaluate:

$(\log_2 9)(\log_9 32)$ (10 marks)

(iii) Compute the value of y by logarithms, arranging your work neatly:

$$y=\sqrt[7]{\frac{(\tan 19^\circ 45')^2 \times (\cos 77^\circ 16')^3}{(3.004)^5 \times (50.06)^3}}$$ (10 marks)

(cont'd.p.2)..

Nour Bassi 45

Algebra. 4th Year Scientific. 29/5/1966.

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4. (i) The eighth term of an arithmetical progression is six times the third term. Find the second term of the progression.

(10 marks).

(ii) An invalid on a certain day was able to take a single step of 18 inches. If he was each day to walk twice as far as on the preceding day, how long would it be before he can take a walk of 512 yards ?

(10 marks)

5. (i) Draw the graph of $y=x^3$ for values of x at half-unit intervals from -2 to 2.2, taking one inch as one unit on the axis of x and 0.4 inch as one unit on the axis of y.

(6 marks)

(ii) Using the same axes and scales, draw another graph to find the roots of the equation $x^3 - \frac{13}{4}x - \frac{3}{2} = 0$.

(7 marks)

(iii) From your diagram, find all values of x which make the expression $\left[x^3-(\frac{13}{4}x+\frac{3}{2})\right]$, positive.

(7 marks).

Naim Shahrabani

Shamash Secondary School
Final Examination, May, 1966.

Subject: Algebra
Class : 4th Year, Scientific.

Date: 29/5/1966
Time: 8:00 - 11:00 a.m.

Answer all f i v e questions:

1. (a) Factor the following:

(i) $my^4 + 16mx^4 - 12mx^2y^2$ (4 marks)

(ii) $a^2b^2x^2 - a^2b^2 - 2abx^2 + 2ab + x^2 - 1$ (4 marks)

(iii) $27x^6y^9 + 64y^3$ (4 marks)

(b) Find the value of p and q which will make the expression $2x^3 + px^2 + qx + 1$ divisible by (x-1) and (x+1), and find the third factor. (8 marks)

2. (a) Use the method of completing the square to show that the sum of the roots of the equation $ax^2+bx+c=o$ is equal to $(-\frac{b}{a})$ and that their product is equal to $(\frac{c}{a})$. (10 marks)

(b) Find the value of x from the following equation:

$$3.10^{2x} - 13.10^{x} + 4 = o$$ (10 marks)

3. Solve only two sections from the following three sections:

(i) Find the value of x from the following equation:

$$\log_3 (2x^2-7x) = 2$$ (10 marks)

(ii) Without using tables evaluate:

$$(\log_2 9)(\log_9 32)$$ (10 marks)

(iii) Compute the value of y by logarithms, arranging your work neatly:

$$y=\sqrt[7]{\frac{(\tan 19^\circ 4\acute{5})^2 \times (\cos 77^\circ 1\acute{6})^3}{(3.004)^5 \times (50.06)^3}}$$ (10 marks)

(cont'd.p.2)..

Algebra. 4th Year Scientific. 29/5/1966.

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4. (i) The eighth term of an arithmetical progression is six times the third term. Find the second term of the progression.

(10 marks).

(ii) An invalid on a certain day was able to take a single step of 18 inches. If he was each day to walk twice as far as on the preceding day, how long would it be before he can take a walk of 512 yards ?

(10 marks)

5. (i) Draw the graph of $y=x^3$ for values of x at half-unit intervals from -2 to 2.2, taking one inch as one unit on the axis of x and 0.4 inch as one unit on the axis of y.

(6 marks)

(ii) Using the same axes and scales, draw another graph to find the roots of the equation $x^3 - \frac{13}{4}x - \frac{3}{2} = 0$.

(7 marks)

(iii) From your diagram, find all values of x which make the expression $\left[x^3 - (\frac{13}{4}x + \frac{3}{2})\right]$, positive.

(7 marks).

Shamash Secondary School
Final Examination, May, 1966.

Subject: Algebra — Date: 29/5/1966
Class : 4th Year, Scientific. — Time: 8:00 - 11:00 a.m.

Answer all f i v e questions:

1. (a) Factor the following:

(i) $my^4 + 16mx^4 - 12mx^2y^2$ (4 marks)

(ii) $a^2b^2x^2 - a^2b^2 - 2abx^2 + 2ab + x^2 - 1$ (4 marks)

(iii) $27x^6y^9 + 64y^3$ (4 marks)

(b) Find the value of p and q which will make the expression $2x^3 + px^2 + qx + 1$ divisible by (x-1) and (x+1), and find the third factor. (8 marks)

2. (a) Use the method of completing the square to show that the sum of the roots of the equation $ax^2+bx+c=o$ is equal to $(-\frac{b}{a})$ and that their product is equal to $(\frac{c}{a})$. (10 marks)

(b) Find the value of x from the following equation:

$3.10^{2x} - 13.10^{x} + 4 = o$ (10 marks)

3. Solve only two sections from the following three sections:

(i) Find the value of x from the following equation:

$\log_3 (2x^2-7x) = 2$ (10 marks)

(ii) Without using tables evaluate:

$(\log_2 9)(\log_9 32)$ (10 marks)

(iii) Compute the value of y by logarithms, arranging your work neatly:

$$y=\sqrt[7]{\frac{(\tan 19^\circ 45')^2 \times (\cos 77^\circ 16')^3}{(3.004)^5 \times (50.06)^3}}$$ (10 marks)

(cont'd.p.2)..

Algebra. 4th Year Scientific. 29/5/1966.

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4. (i) The eighth term of an arithmetical progression is six times the third term. Find the second term of the progression.

(10 marks).

(ii) An invalid on a certain day was able to take a single step of 18 inches. If he was each day to walk twice as far as on the preceding day, how long would it be before he can take a walk of 512 yards ?

(10 marks)

5. (i) Draw the graph of $y=x^3$ for values of x at half-unit intervals from -2 to 2.2, taking one inch as one unit on the axis of x and 0.4 inch as one unit on the axis of y.

(6 marks)

(ii) Using the same axes and scales, draw another graph to find the roots of the equation $x^3 - \frac{13}{4}x - \frac{3}{2} = 0$.

(7 marks)

(iii) From your diagram, find all values of x which make the expression $\left[x^3-(\frac{13}{4}x+\frac{3}{2})\right]$, positive.

(7 marks).

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Shamash Secondary School
Final Examination, May, 1966.

Subject: Algebra — Date: 29/5/1966
Class : 4th Year, Scientific. — Time: 8:00 - 11:00 a.m.

Answer all f i v e questions:

1. (a) Factor the following:

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(iii) $27x^6y^9 + 64y^3$ (4 marks)

(b) Find the value of p and q which will make the expression $2x^3 + px^2 + qx + 1$ divisible by (x-1) and (x+1), and find the third factor. (8 marks)

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(b) Find the value of x from the following equation:

$3 \times 10^{2x} - 13 \times 10^{x} + 4 = o$ (10 marks)

3. Solve only two sections from the following three sections:

(i) Find the value of x from the following equation:

$$\log_3 (2x^2-7x) = 2$$ (10 marks)

(ii) Without using tables evaluate:

$$(\log_2 9)(\log_9 32)$$ (10 marks)

(iii) Compute the value of y by logarithms, arranging your work neatly:

$$y = \sqrt[7]{\frac{(\tan 19^\circ 45')^2 \times (\cos 77^\circ 16')^3}{(3.004)^5 \times (50.06)^3}}$$ (10 marks)

(cont'd.p.2)..

Wadia School.

Algebra. 4th Year Scientific. 29/5/1966.

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4. (i) The eighth term of an arithmetical progression is six times the third term. Find the second term of the progression.

(10 marks).

(ii) An invalid on a certain day was able to take a single step of 18 inches. If he was each day to walk twice as far as on the preceding day, how long would it be before he can take a walk of 512 yards ?

(10 marks)

5. (i) Draw the graph of $y=x^3$ for values of x at half-unit intervals from -2 to 2.2, taking one inch as one unit on the axis of x and 0.4 inch as one unit on the axis of y.

(6 marks)

(ii) Using the same axes and scales, draw another graph to find the roots of the equation $x^3 - \frac{13}{4}x - \frac{3}{2} = 0$.

(7 marks)

(iii) From your diagram, find all values of x which make the expression $\left[x^3-(\frac{13}{4}x+\frac{3}{2})\right]$, positive.

(7 marks).

Shamash Secondary School
Final Examination, May, 1966.

Subject: Algebra
Class : 4th Year, Scientific.

Date: 29/5/1966
Time: 8:00 - 11:00 a.m.

Answer all f i v e questions:

1. (a) Factor the following:

(i) $my^4 + 16mx^4 - 12mx^2y^2$ (4 marks)

(ii) $a^2b^2x^2 - a^2b^2 - 2abx^2 + 2ab + x^2 - 1$ (4 marks)

(iii) $27x^6y^9 + 64y^3$ (4 marks)

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(cont'd.p.2)..

Algebra. 4th Year Scientific. 29/5/1966.

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4. (i) The eighth term of an arithmetical progression is six times the third term. Find the second term of the progression.

(10 marks).

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(6 marks)

(ii) Using the same axes and scales, draw another graph to find the roots of the equation $x^3 - \frac{13}{4}x - \frac{3}{2} = 0$.

(7 marks)

(iii) From your diagram, find all values of x which make the expression $\left[x^3-(\frac{13}{4}x+\frac{3}{2})\right]$, positive.

(7 marks).

Solutions to Conditional Exam in Algebra

Sept., 1965, 4th year

1. (i) $2(x^2-2)^2+5(x^2-2)-12=0$ or $[2(x^2-2)-3][(x^2-2)+4]=0$

$\therefore 2(x^2-2)-3=0$ or $2x^2=7$ $\therefore x=\pm\sqrt{\frac{7}{2}}$ or $x=\pm\frac{1}{2}\sqrt{14}$ Ans. I, II

also $(x^2-2)+4=0$ or $x^2=-2$ $\therefore x=\pm\sqrt{-2}=\pm\sqrt{2}\,i$ Ans. III + IV

(ii) (a) $\left\{\frac{(9^{n+\frac{1}{4}})(\sqrt{3.3^n})}{3\sqrt{3^{-n}}}\right\}^{\frac{1}{n}}=\left\{\frac{(3^2)^{\frac{4n+1}{4}}\times(3)^{\frac{n+1}{2}}}{3^{1-\frac{n}{2}}}\right\}^{\frac{1}{n}}=\left\{\frac{3^{\frac{4n+1}{2}}\cdot 3^{\frac{n+1}{2}}}{3^{\frac{2-n}{2}}}\right\}^{\frac{1}{n}}=$ (3 marks)

$=\left\{3^{\frac{4n+1+n+1+n-2}{2}}\right\}^{\frac{1}{n}}=(3^{3n})^{\frac{1}{n}}=3^3=27$ Ans.

(6 marks)

(b) $\frac{6x^2y^2}{m+n}\div\left[\frac{3(m-n)x}{7(r+s)}\div\left\{\frac{4(r-s)}{21xy^2}\div\frac{r^2-s^2}{4(m^2-n^2)}\right\}\right]=$

$=\frac{6x^2y^2}{m+n}\div\left[\frac{3(m-n)x}{7(r+s)}\div\frac{\{4(r-s)\}\{4(m^2-n^2)\}}{21xy^2(r^2-s^2)}\right]=$

$=\frac{6x^2y^2}{m+n}\div\frac{[3(m-n)x][21xy^2(r^2-s^2)]}{[7(r+s)][16(r-s)(m^2-n^2)]}$

$=\frac{6x^2y^2}{m+n}\times\frac{7\times16(m^2-n^2)(r^2-s^2)}{3\times21(m-n)(r^2+s^2)x^2y^2}=\frac{6\times7\times16}{3\times21}=\frac{32}{3}$ Ans.

(6 marks)

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2. (i) $\log_{10}y=a+b\log_{10}x$ } when $x=1$, $y=1000$ } $x=10$
$\quad$ " $x=0.1$, $y=100$ } $y=?$

$\therefore \log_{10}1000=a+b\log_{10}1$ or $3=a$ or $a=3$

$\log_{10}100=a+b\log_{10}0.1$ or $2=a-b$ or $b=1$

$\therefore \log_{10}y=3+\log_{10}x$ $\therefore \log_{10}y=3+\log_{10}10$ or $\log_{10}y=4$

$\therefore y=10^4$ Ans. (10 marks)

(ii) $\log_{10}2=0.301030$, $\log_{10}1.005=0.002166$, $\log_{10}402=?$, $\log_{10}0.0804=?$

$1.005=\frac{1005}{1000}=\frac{5\times3\times67}{1000}=\frac{3\times67}{200}$ $\therefore 3\times67=200(1.005)$

$402=2\times3\times67=2[200(1.005)]=2^2\times100\times1.005$

$0.0804=\frac{804}{10^4}=\frac{2\times402}{10^4}=\frac{2^3\times100\times1.005}{10^4}=\frac{2^3\times1.005}{100}$

2 (ii) cont.

$\therefore \log_{10} 402 = 2\log_{10} 2 + 2 + \log_{10} 1.005 = 2 \times 0.301030 + 2 + 0.002166$

$= 0.602060 + 2 + 0.002166 = 2.604226$ Ans. (5 marks)

$\therefore \log 0.804 = 3\log_{10} 2 + \log_{10} 1.005 - 2 = 3 \times 0.301030 + 0.002166 - 2$

$= 0.903090 + 0.002166 - 2 = 0.905256 - 2$

$= -1.094744$ Ans. (5 marks)

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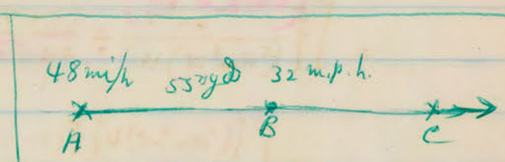
3. (i) Let the time for train A to overtake train B at pt. C be t seconds

$\therefore$ dist AC $= 48\left(\frac{t}{3600}\right)$ mils
$\therefore$ dist BC $= 32\left(\frac{t}{3600}\right)$ mils



$\therefore 48\left(\frac{t}{3600}\right) - 32\left(\frac{t}{3600}\right) = \frac{550}{1760}$

$\therefore \frac{16t}{3600} = \frac{55}{176}$ or $t = \frac{55}{176} \times \frac{3600}{16} = \frac{5}{16} \times \frac{450}{2} = \frac{1125}{16}$

$= 70.3$ Sec $=$ 70 Sec to the nearest second (7 marks) Ans.

(ii) $x\left(\frac{t}{3600}\right) - y\left(\frac{t}{3600}\right) = \frac{D}{1760}$ $\therefore t\left(\frac{x-y}{3600}\right) = \frac{D}{176}$

$\therefore t = \frac{360D}{176(x-y)}$ or $t = \frac{45D}{22(x-y)}$ Ans. (7 marks)

(iii) From the above formula $x - y = \frac{45D}{22t}$

$\therefore y = x - \frac{45D}{22t}$ Ans. (6 marks)

Solutions to Conditional Exam in Algebra Cont.

Sept., 1965

4. (i) the first number which is divisible by 13 + greater than 60 is $65 = (5\times13)$ and the last number divisible by 13 and less than 600 is $598 = (46\times13)$ ∴ we a have an Arith. Prog. in which $a = 65$, $l = 598$, $d = 13$. to find n + S.

$l = a + (n-1)d$ ∴ $598 = 65 + (n-1)13$ ∴ $13(n-1) = 533$

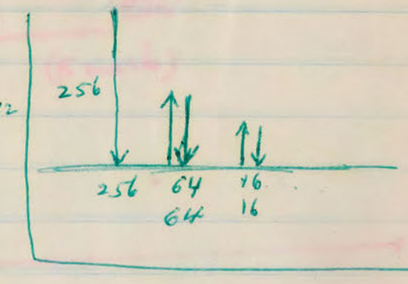
∴ $n - 1 = \frac{533}{13} = 41$ ∴ $n = 42$

∴ $S = \frac{n}{2}(a+l)$ or $S = \frac{42}{2}(65+598)$ or $S = 21 \times 663$

or $S = 13923$ Ans. (10 marks)

(ii) ① total at the end of 5th time

is $= 256 + 64\times2 + 16\times2 + 4\times2 + 1\times2$



$S = 256 + 2(64+16+4+1)$

$= 256 + 2\left[\frac{64\{1-(\frac{1}{4})^4\}}{1-\frac{1}{4}}\right]$

$= 256 + 2\left[\frac{4\times64}{3}\left(1-\frac{1}{4^4}\right)\right] = 256 + \frac{512}{3}\left(1-\frac{1}{256}\right)$

$= 256 + \frac{512}{3} \times \frac{255}{256} = 256 + 170 = 426$ Ans. (5 marks)

② total dist. at the end of n^{th} strike is

$S' = 256 + 2(64+16+4+\ldots$ to $(n-1)$ terms)

$S' = 256 + 2\left[\frac{64\{1-(\frac{1}{4})^{n-1}\}}{1-\frac{1}{4}}\right]$ Ans. $= 256 + 2\left[\frac{4\times64}{3}\left(1-\frac{1}{4^{n-1}}\right)\right]$

$= 256 + \frac{512}{3}\left(1-\frac{1}{4^{n-1}}\right) = 256 + \frac{512}{3} - \frac{512}{3\times4^{n-1}}$

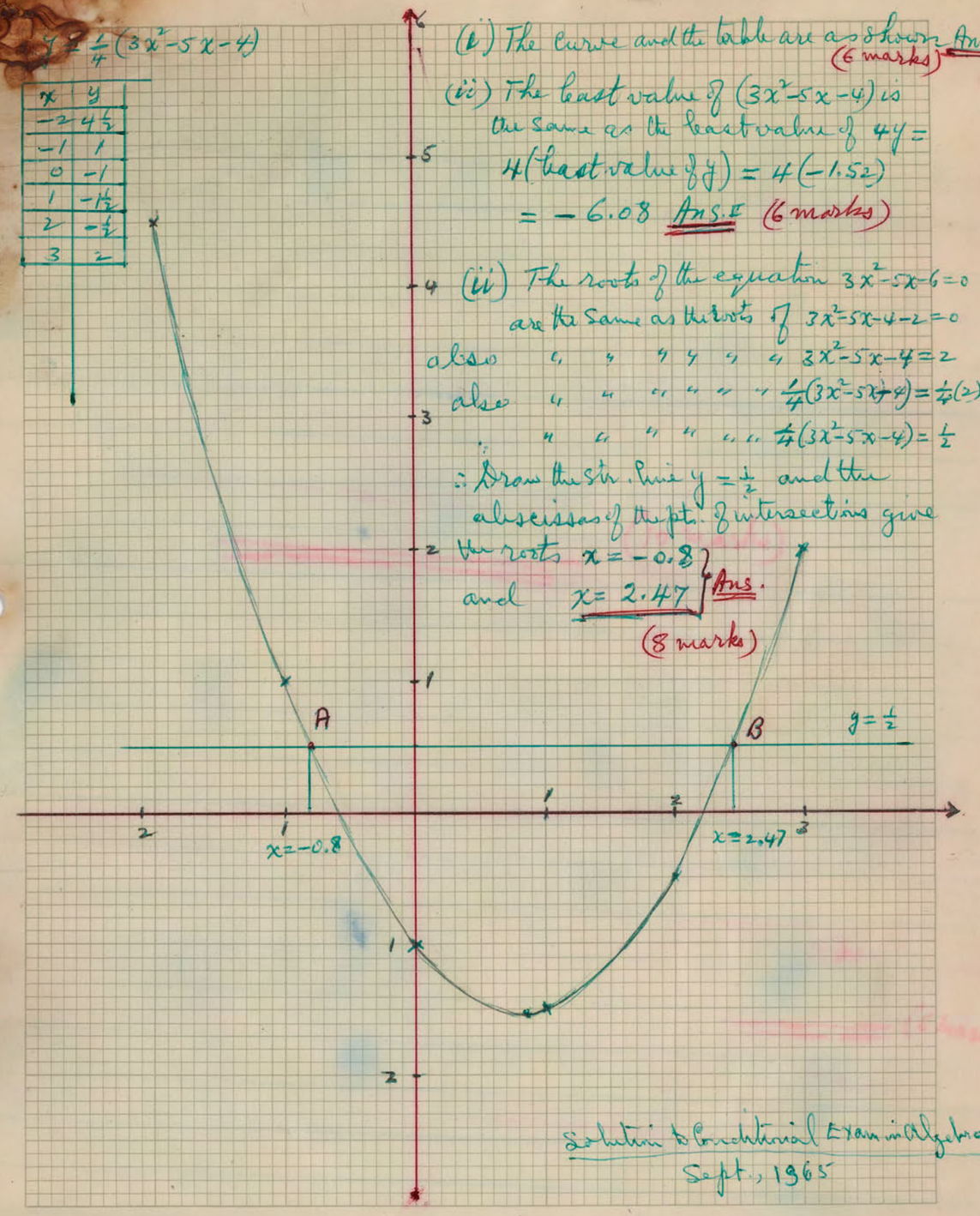
$= \frac{768+512}{3} - \frac{512}{3\times4^{n-1}} = \frac{1280}{3} - \frac{512}{3\times4^{n-1}}$

$= \frac{1280}{3} - \frac{2^9}{3\times2^{2n-2}} = \frac{1280}{3} - \frac{2^{11-2n}}{3} = \frac{1280-2^{11-2n}}{3}$

Ans (5 marks)

$y = \frac{1}{4}(3x^2 - 5x - 4)$

x	y
−2	$4\frac{1}{2}$
−1	1
0	−1
1	$-1\frac{1}{2}$
2	$-\frac{1}{2}$
3	2



(i) The curve and the table are as shown Ans. (6 marks)

(ii) The least value of $(3x^2 - 5x - 4)$ is the same as the least value of $4y =$
4(least value of y) $= 4(-1.5\frac{1}{2})$
$= -6.08$ Ans. II (6 marks)

(iii) The roots of the equation $3x^2 - 5x - 6 = 0$
are the same as the roots of $3x^2 - 5x - 4 - 2 = 0$
also " " " " " " $3x^2 - 5x - 4 = 2$
also " " " " " " $\frac{1}{4}(3x^2 - 5x - 4) = \frac{1}{4}(2)$
" " " " " " $\frac{1}{4}(3x^2 - 5x - 4) = \frac{1}{2}$

∴ Draw the str. line $y = \frac{1}{2}$ and the abscissas of the pts. of intersections give the roots $x = -0.8$ and $x = 2.47$ Ans. (8 marks)

Solution to Continuous Exam in Algebra
Sept., 1965

Shamash Secondary School
Conditional Examination, Sept.1965

Subject: Algebra
Class: 4th Year, Scientific

Date: 16/9/1965
Time: 8:00-11:00

Attempt all questions:

1. (i) Solve by the shortest possible way the following equation:

$$2(x^2-2)^2 + 5(x^2-2) - 12 = 0$$

(8 marks)

Give your answers correct to two decimal places using tables if necessary.

(ii) Reduce to simplest form the expressions:

(a) $$\left[\frac{(9^{n+\frac{1}{4}})(\sqrt{3.3^n})}{3\sqrt{3^{-n}}}\right]^{\frac{1}{n}}$$ (6 marks)

(b) $$\frac{6x^2y^2}{m+n} \div \left[\frac{3(m-n)x}{7(r+s)} \div \left\{\frac{4(r-s)}{21xy^2} \div \frac{r^2-s^2}{4(m^2-n^2)}\right\}\right]$$

(6 marks)

2. (i) The variables x and y are related by the equation $\log_{10} y = a + b\log_{10} x$ where a and b are constants. If y = 1000 when x=1 and y=100 when x =0.1, find the value of y when x=10

(10 marks)

(ii) Given that $\log_{10} 2 = 0.301030$ and $\log_{10} 1.005 = 0.002166$. Calculate, without the use of tables: (a) $\log_{10} 402$, (b) $\log_{10} 0.0804$

(10 marks)

3. (i) A car travelling steadily at 48 m.p.h. is 550 yd. behind a car travelling steadily at 32 m.p.h. Find, to the nearest second, the time taken by the faster car to overtake the slower.

(ii) If the faster car is travelling at x m.p.h. and the slower at y m.p.h. and D yds. is the distance between them, find a formula for the time, t sec., taken to overtake.

(iii) From your formula express y in terms of the other letters.

(20 marks)

(p.2..)

Algebra 4th Year. Scientific. 16/9/1965

4. (i) Find the sum of all the numbers between 60 and 600 which are exactly divisible by 13.

(10 marks)

(ii) An elastic ball is dropped on to a horizontal smooth plate and allowed to go on bouncing in the same vertical line. At each bounce it rises to one-fourth of the height from which it has fallen. If it is originally released from a height of 256 ft., find the total distance the ball has moved altogether, up and down, by the time it strikes the plate for: (1) the 5th time (2) the nth time.

(10 marks)

5. (i) Draw the graph of $\frac{1}{4}(3x^2-5x-4)$ for values of x from -2 to +3, using a scale of 1 inch to 1 unit on each axis.

(6 marks)

(ii) Use your graph to find the least value of $3x^2-5x-4$.

(6 marks)

(iii) By drawing the appropriate straight line on your graph, solve the equation $3x^2-5x-6=0$

(8 marks)

الرقم :

الاسم :

Monthly Examination, August 1965

Subject: General Mathematics
Class: 4th Year Secondary

Date: 19/8/1965
Time: 7:30-8:15 a.m.

I. Give the English Equivalent of the following:

١ــ ارقـــام
٢ــ مراتـــب
٣ــ الطرح
٤ــ العوامل
٥ــ أس القوة
٦ــ المقسوم عليه
٧ــ ناتج القسمة
٨ــ المقســـوم
٩ــ باقي القسمة
١٠ــ مضاعـــف

١١ــ اعداد زوجية متتالية
١٢ــ اعداد فردية متتالية
١٣ــ اعداد اولية
١٤ــ الجزء الصحيح من العدد
١٥ــ المقام المشترك الاصغر
١٦ــ كسر لفظي
١٧ــ مقلوب العدد
١٨ــ الكسور العشرية المنتهية
١٩ــ الكسور العشرية الدورية
٢٠ــ بسط الكسر
٢١ــ مقام الكسر
٢٢ــ الخطأ المئوى
٢٣ــ النسبة والتناسب
٢٤ــ الوسط المتناسب بين عددين
٢٥ــ ربح المساهم (ربح عامل الاسهم)
٢٦ــ البديهية
٢٧ــ الموضوعة
٢٨ــ زاوية حادة
٢٩ــ زاوية منفرجة
٣٠ــ زاوية منعكسة
٣١ــ قطعة دائرة
٣٢ــ قطاع دائرة
٣٣ــ المعاليـــم
٣٤ــ المجاهيـــل
٣٥ــ زاويتان متتامتان

ـــ يتبع ـــ

٣٦ــ زاويتان متكاملتان

٣٧ــ مضلع متساوى الاضلاع

٣٨ــ مثلث متساوى الساقين

٣٩ــ الخطوط المتوسطة في المثلث

٤٠ــ المعين

٤١ــ المحل الهندسي

٤٢ــ المستقيم القاطع للدائرة

٤٣ــ ازالة وادخال الاقواس

٤٤ــ نقل حدود المعادلة من جهة الى الجهة الاخرى

٤٥ــ متطابقة

٤٦ــ متباينة

٤٧ــ مقدار جبرى متجانس

٤٨ــ درجة المقدار الجبرى

٤٩ــ المعامل الحرفي

٥٠ــ مقدار جبرى من الدرجة الثانية.

(75 marks)

II. Fill in the blanks in the following equations:-

1. one furlong = ( ) chains = ( ) mile
2. one chain = ( ) yards = ( ) links
3. one statute mile = ( ) yds.=( ) ft.
4. one nautical mile = ( ) ft.
5. one sq. chain = ( ) sq. yds.
6. one acre = ( ) sq. ch. = ( ) sq. yds.
7. one gallon = ( ) pints
8. one bushel = ( ) gallons = ( ) pecks
9. one English ton = ( ) lbs.⌒( ) kilograms
10. one English ton = ( ) cwt. = ( ) qr.= ( ) stones.

(25 marks).

الاسم :

الرقم :

Monthly Examination, August 1965

Subject: General Mathematics — Date: 19/8/1965
Class: 4th Year Secondary — Time: 7:30-8:15 a.m.

I. Give the English Equivalent of the following:

١ــ ارقـــام
٢ــ مراتـــب
٣ــ الطرح
٤ــ العوامل
٥ــ أس القوة
٦ــ المقسوم عليه
٧ــ ناتج القسمة
٨ــ المقســـوم
٩ــ باقي القسمة
١٠ــ مضاعـــف

١١ــ اعداد زوجية متتالية
١٢ــ اعداد فردية متتالية
١٣ــ اعداد اولية
١٤ــ الجزء الصحيح من العدد
١٥ــ المقام المشترك الاصغر
١٦ــ كسر لفظي
١٧ــ مقلوب العدد
١٨ــ الكسور العشرية المنتهية
١٩ــ الكسور العشرية الدورية
٢٠ــ بسط الكسر
٢١ــ مقام الكسر
٢٢ــ الخطأ المئوى
٢٣ــ النسبة والتناسب
٢٤ــ الوسط المتناسب بين عددين
٢٥ــ ربح المساهم (ربح عامل الاسهم)
٢٦ــ البديهية
٢٧ــ الموضوعة
٢٨ــ زاوية حادة
٢٩ــ زاوية منفرجة
٣٠ــ زاوية منعكسة
٣١ــ قطعة دائرة
٣٢ــ قطاع دائرة
٣٣ــ المعالـــيم
٣٤ــ المجاهيـــل
٣٥ــ زاويتان متتامتان

ـــ يتبع ـــ

الرقم:
الاسم:

٣٦ــ زاويتان متكاملتان
٣٧ــ مضلع متساوى الاضلاع
٣٨ــ مثلث متساوى الساقين
٣٩ــ الخطوط المتوسطة في المثلث
٤٠ــ المحين
٤١ــ المحل الهندسي
٤٢ــ المستقيم القاطع للدائرة
٤٣ــ ازالة وادخال الاقواس
٤٤ــ نقل حدود المعادلة من جهة الى الجهة الاخرى
٤٥ــ متطابقة
٤٦ــ متباينة
٤٧ــ مقدار جبرى متجانس
٤٨ــ درجة المقدار الجبرى
٤٩ــ المعامل الحرفي
٥٠ــ مقدار جبرى من الدرجة الثانية.

(75 marks)

II. Fill in the blanks in the following equations:-

1. one furlong = ( ) chains = ( ) mile
2. one chain = ( ) yards = ( ) links
3. one statute mile = ( ) yds.=( ) ft.
4. one nautical mile = ( ) ft.
5. one sq. chain = ( ) sq. yds.
6. one acre = ( ) sq. ch. = ( ) sq. yds.
7. one gallon = ( ) pints
8. one bushel = ( ) gallons = ( ) pecks
9. one English ton = ( ) lbs.≏( ) kilograms
10. one English ton = ( ) cwt. = ( ) qr.= ( ) stones.

(25 marks).

SHAMASH SECONDARY SCHOOL
FINAL EXAMINATION, JUNE, 1965.

Subject : Algebra. Date : 1/6/1965.
Class : 4th year, secondary, sections A & B. Time : 8:00-11:00 a.m.

Attempt all questions :

1. (i) If $m = \frac{2x + y}{x + 2y}$, find an expression for y in terms of m and x.

If also $Y = mx$, find the values of m. (7 marks).

2. (ii) Resolve into two factors : $c^3 - 27b^3 + a^3 + 9abc$ (7 marks).

(iii) Resolve the expression $5x^2 - 14x + 9$ into two factors and show that the value of this expression is negative when x lies between 1 and 1.8. (6 marks).

3. (i) Compute by logarithms, arranging your work neatly :

$$\sqrt[7]{\frac{(\cos^2 18^\circ\ 47')(\sin^3 48^\circ\ 21')}{(10.09)^3\ (0.000\,2049)}}$$ (6 marks).

(ii) If $2 \log a - 5 \log b = 3 \log c$, find 'a' in terms of 'b' and 'c'. (4 marks).

(iii) Given $\log_a 4.41 = 2$, calculate the value of 'a'. (4 marks).

(iv) Solve the equation $2^{3-x} = 3^{2x+1}$ giving your answer correct to three decimal places. (6 marks).

4. (i) Write down and simplify an expression for the nth term of the arithmetic progression 3 , 7 , 11 , ... (4 marks).

If the sum of n terms of this progression is $bn + cn^2$ find the values of b and c and the sum of the first thirty terms. (8 marks).

(ii) The product of the first and seventh terms of a geometric progression is equal to the fourth term; andthe sum of the first and fourth terms is 9. Find the sum of the first seven terms of the progression. (8 marks).

5. (i) Draw the graph of $y = (x - 1)(x - 3)^2$ for values of x from -½ to 5, choosing 0.5 inch for your unit on the x-axix and 0.2 inch for your unit on the y-axix. To get a good drawing of the curve, choose successive values of x at intervals of halves, beginning with -½. (5 marks).

(ii) From this graph find an approximate maximum value and an exact minimum value for y and the corresponding values of x which make y a maximum or a minimum. (5 marks).

(iii) By plotting another graph on the same diagram find the roots of the equation $(x - 1)(x - 3)^2 = 5x - 9$. (5 marks).

(iv) From these two graphs find the values of x for which the function $(x - 1)(x - 3)^2$ is always greater than $(5x - 9)$. (5 marks).

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2. 'A' can walk a mile [illegible] less time than B would take. In a walking race, B has a start of ¼ mile and A overtakes B in 10 minutes. Assuming both men walk at a uniform rate, find their rates of walking in miles per hour. (20 marks).

Shamsiah Secondary School (1)

Final Examination, June 1965

Subject : Algebra — Date : 1/6/1965

Class : 4th Scientific, sections A + B — Time : 8:00 – 11:00 a.m.

Attempt all questions

I. (i) If $\frac{2x+y}{x+2y} = m$, find an expression for y in terms of m and x.

If also $y = mx$, find the value of m. (7 marks)

(ii) Resolve into two factors: ~~[illegible]~~ $c^3 - 27b^3 + a^3 + 9abc$ (7 marks)

(iii) Resolve the expression $5x^2 - 14x + 9$ into two factors and show that the value of this expression is negative when x lies between 1 and 1·8 (6 marks)

II. A man can walk a mile in 2 minutes less time than B would take. In a walking race, B has a start of ¼ mile and A overtakes B in 10 minutes. Assuming both men walk at a uniform rate, find their rates of walking in miles per hour. (20 marks)

III. (i) Compute by logarithms arranging your work neatly:

$$\sqrt[7]{\frac{\cos^2 18^\circ 47' \times \sin^3 48^\circ 21'}{(10.09)^3 \times 0.0003043}}$$ (6 marks)

(ii) If $2\log a - 5\log b = 3\log c$, find (a) in terms of b and c. (4 marks)

(iii) Given $\log_a 4.41 = 2$, calculate the value of a. (4 marks)

(IV). Solve the equation $2^{3-x} = 3^{2x+1}$ giving your answer correct to 2 decimal places (6 marks)

IV. (i) Write down and simplify an expression for the n^{th} term of the Arithmetic progression 3, 7, 11, ….. (4 marks)

If the sum of n terms of this progression is $bn + cn^2$, find the values of b and c and the sum of the first thirty terms. (8 marks)

(ii) The product of the first and seventh terms of a geometric progression is equal to the fourth term, and the sum of the first & fourth terms is 9. Find the sum of the first seven terms of the progression. (8 marks)

I. (i) Draw the graph of $y = (x-\frac{1}{2})(x-3)^2$ for values of x from $-\frac{1}{2}$ to 5, choosing 0.5 inch for your unit on the x-axis and 0.2 inches for your unit on the y-axis. To get a good drawing of the curve, choose successive values of x at intervals of halves, beginning with $-\frac{1}{2}$.
(5 marks)

(ii) From this graph find an approximate maximum value and the exact minimum value for y and the corresponding values of x which make y a maximum or a minimum. (5 marks)

(iii) By plotting another graph on the same diagram find the roots of the equation $(x-1)(x-3)^2 = 5x-9$. (5 marks)

(iv) From these two graphs find the values of x for which the function $(x-1)(x-3)^2$ is always greater than $(5x-9)$. (5 marks)

Solutions to Algebra Exam. Final exam Fourth year, 1965

1. (i) $\frac{2x+y}{x+2y} = m \quad \therefore 2x+y = mx+2my \quad \therefore y(2m-1) = x(2-m) \quad \therefore y = \frac{x(2-m)}{2m-1}$ Ans.1

If $y = mx$ then $mx = \frac{x(2-m)}{2m-1} \quad \therefore m(2m-1) = 2-m$ or $2m^2 = 2 \quad \therefore m^2 = 1 \quad \therefore m = \pm 1$ Ans.

(ii) $c^3 - 27b^3 + a^3 + 9abc = a^3 + (-3b)^3 + a^2 - 3a(-3b)c$

$= (a-3b+c)(a^2+9b^2+c^2+3ab-ac+3bc)$ Ans.

(iii) $5x^2 - 14x + 9$ into two factors $\therefore 5x^2 - 14x + 9 = (5x-9)(x-1)$ Ans.1

$5x^2 - 14x + 9 = (5x-9)(x-1) = 5\left(x - \frac{9}{5}\right)(x-1) = 5(x-1.8)(x-1)$

when $1 < x < 1.8$ the factor $(x-1.8) = (-)$ and the factor $(x-1) = (+)$

$\therefore$ the product $= (-) \times (+) = (-)$ Q.E.D

2. Let A's rate of walking $= x$ miles/hr
B's " " " $= y$ miles/hr

$\therefore \frac{1}{y} \times 60 - \frac{1}{x} \times 60 = 2$

since $\frac{60}{y}$ = No. of minutes for B to cover 1 mile

and $\frac{60}{x}$ = " " " A " " 1 "

$\therefore \frac{60}{y} - \frac{60}{x} = 2 \quad \ldots\ldots (1)$

B $\frac{1}{2}$ mile B

Also from the figure: distance $AC = \frac{10}{60}x = \frac{x}{6}$ miles

" $BC = \frac{10}{60}y = \frac{y}{6}$ "

But $AC - BC = \frac{1}{2}$ $\therefore \frac{x}{6} - \frac{y}{6} = \frac{1}{2}$

$x - y = 3$ or $y = \frac{x-3}{2} \quad \ldots\ldots (2)$

from eq. (1) $30x - 30y = xy \quad \ldots\ldots (3)$ and from (2) we obtain

$30x - 30\left(\frac{x-3}{2}\right) = x\left(\frac{x-3}{2}\right)$ or $60x - 60x + 90 = x^2 - 3x$ or

$-x^2 - 3x - 90 = 0$ or $(x-15)(x+6) = 0 \quad \therefore x = -6$ inadmissible

or $x = \frac{15}{2} = 7.5$ miles/hr. and from eq. (2) $y = \frac{x-3}{2} = \frac{15-3}{2} = 6$

$\therefore$ $y = 6$ miles/hr. Ans.

3. (i) Let $x = \sqrt[7]{\dfrac{\cos^2 18°47' \times \sin^3 48°21'}{(0.09)^3 \times 0.000\,2049}}$ (6)

$\log \cos 18°47' = \bar{1}.9762$
$\log \sin 48°21' = \bar{1}.8734$
$\log 10.09 = 1.0037$
$\log 0.0002049 = \bar{4}.3115$

or $\cos 18°47' = 0.9465$
$\therefore \log \cos 18°47' = \bar{1}.9761$
$\sin 48°21' = 0.7472$
$\therefore \log \sin 48°21' = \bar{1}.8734$

$\therefore 2\log \cos 18°47' = \bar{1}.9522$
$3\log \sin 48°21' = \bar{1}.6202$ (+)
$\log$ Num. $= \bar{1}.5724$
$\log$ Den. $= \bar{1}.3226$ (−)
$7\log x = 0.2498$
$\log x = 0.03568 =$
$\therefore \log x = 0.0357$ Correct to 4 dec. pl.
$\therefore x = 1.086$ Ans.

$2\log \cos 18°47' = \bar{1}.9524$
$3\log \sin 48°21' = \bar{1}.6202$ (+)
$\log$ Num. $= \bar{1}.5726$
$\log$ Den. $= \bar{1}.3226$ (−)
$7\log x = 0.2500$
$\log x = 0.03571 = 0.0357$ Correct to 4 dec. pl.
$x = 1.086$ Ans.

$3\log 10.09 = 3.0111$
$\log 0.0002049 = \bar{4}.3115$ (−)
$\log$ Den. $= \bar{1}.3226$

(4)

(ii) $2\log a - 5\log b = 3\log c$, $a = ?$
$\log a^2 - \log b^5 = \log c^3$ or $\log \frac{a^2}{b^5} = \log c^3$
$\therefore \frac{a^2}{b^5} = c^3 \therefore a^2 = b^5 \cdot c^3 \therefore a = b^{\frac{5}{2}} c^{\frac{3}{2}}$ Ans.

(4)

(iii) $\log_a 4.41 = 2$, $a = ?$
$\therefore 4.41 = a^2 \therefore a = \sqrt{4.41} = 2.1$ Ans.

(7)

(iv) $2^{3-x} = 3^{2x+1} \therefore (3-x)\log 2 = (2x+1)\log 3$
$\therefore 3\log 2 - x\log 2 = 2x\log 3 + \log 3 \therefore x(2\log 3 + \log 2) = 3\log 2 - \log 3$
$\therefore x = \dfrac{3\log 2 - \log 3}{2\log 3 + \log 2} = \dfrac{\log 8 - \log 3}{\log 9 + \log 2} = \dfrac{0.9031 - 0.4771}{0.9542 + 0.3010} = \dfrac{0.4260}{1.2552}$
$= 0.3393\ldots = 0.339$ Correct to 3 dec. pl.
Ans.

4. (i) (1) A.P. 3, 7, 11, ... here $a=3$, $d=4$, No. of terms $=n$, $t_n=?$

$t_n = a+(n-1)d \therefore t_n = 3+(n-1)\times 4 = 4n-1 \therefore t_n = 4n-1$ Ans.1

(2) $S_n = bn+cn^2$ Let $n=1 \therefore S_1 = 3 = b+c$ or $b+c=3$ ... (1)

also let $n=2 \therefore S_2 = 3+7 = 10 = 2b+c(2)^2$ or $2b+4c=10$ ... (2)

or $b+2c=5$ ... (3)

Combining eq. (1) & (3), we get $c=2 \therefore b=1$ Ans.

$\therefore S_{30} = bn+cn^2 = 1\times 30 + 2\times(30)^2 = 30+1800 = 1830$ Ans.3

(ii) In a G.P., we have $t_1 \times t_7 = t_4^2$ also $t_1+t_4=9$, $S_7=?$

Let the first term be a & let the Common Ratio be r

$\therefore (a)(ar^6) = ar^3$ or $a^2r^6 = ar^3$ or $a^2r^3 = 1$ ... (1)

also $a+ar^3 = 9$ ... (2) $\therefore a+1 = 9 \therefore a=8 \therefore r^3 = \frac{1}{8} \therefore r = \frac{1}{2}$

$\therefore a=8$, $r=\frac{1}{2}$, $S_7=?$

$S_7 = \frac{a(1-r^7)}{1-r} = \frac{8\left(1-\left(\frac{1}{2}\right)^7\right)}{1-\frac{1}{2}} = 16\left[1-\frac{1}{128}\right] = 16\left[\frac{128-1}{128}\right]$

$\therefore S_7 = \frac{16\times 127}{128} = \frac{127}{8} = 15\frac{7}{8}$ Ans.

SHAMASH SECONDARY SCHOOL
FINAL EXAMINATION, JUNE, 1965.

Subject : Algebra. Date : 1/6/1965.
Class : 4th year, secondary, sections A & B. Time : 8:00-11:00 a.m.

Attempt all questions :

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5. (i) Draw the graph of $y = (x - 1)(x - 3)^2$ for values of x from -½ to 5, choosing 0.5 inch for your unit on the x-axix and 0.2 inch for your unit on the y-axix. To get a good drawing of the curve, choose successive values of x at intervals of halves, beginning with -½. (5 marks).

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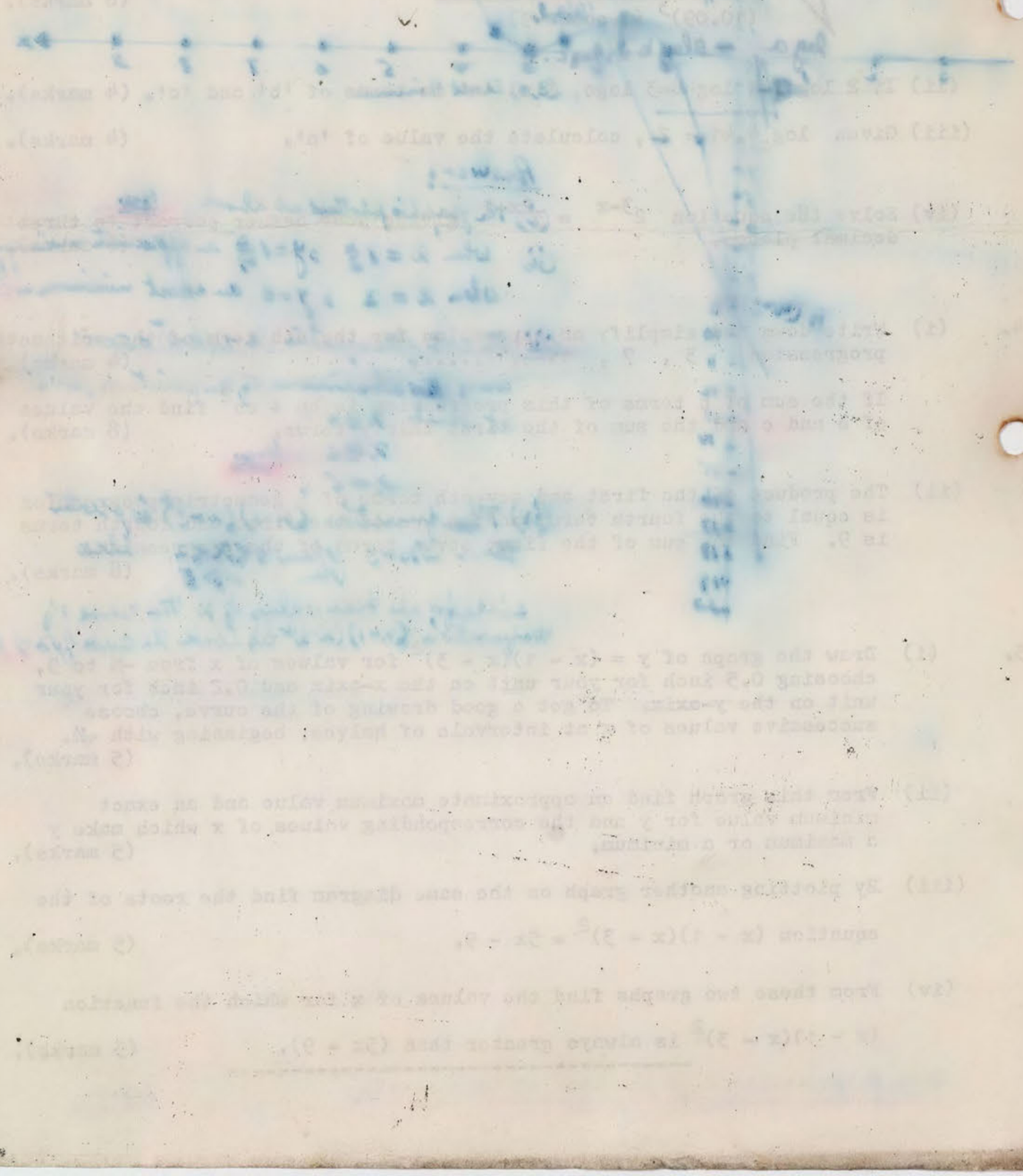
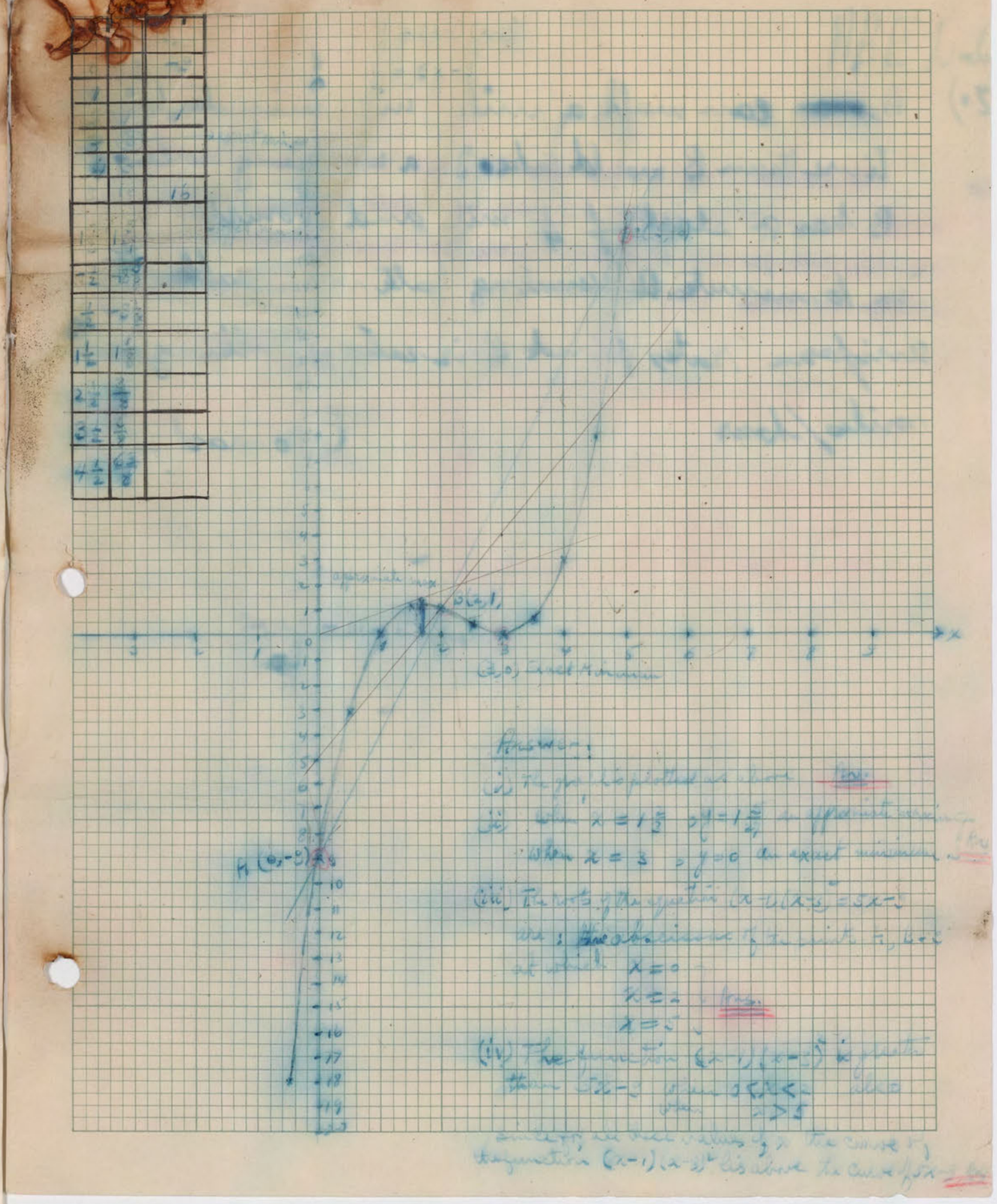
(20) A ——— can walk a mile in 2 minutes less time than B would take. In a walking race, B has a start of $\frac{1}{4}$ mile and A overtakes B in 10 minutes. Assuming both men walk at a uniform rate, find their rate of walking in miles/hour

(10 marks)





SHAMASH SECONDARY SCHOOL
FINAL EXAMINATION, JUNE, 1965.

Subject : Algebra. Date : 1/6/1965.
Class : 4th year, secondary, sections A & B. Time : 8:00-11:00 a.m.

Attempt all questions :

1. (i) If $m = \frac{2x + y}{x + 2y}$, find an expression for y in terms of m and x.

If also $Y = mx$, find the values of m. (7 marks).

2. (ii) Resolve into two factors : $c^3 - 27b^3 + a^3 + 9abc$ (7 marks).

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5. (i) Draw the graph of $y = (x - 1)(x - 3)^2$ for values of x from -½ to 5, choosing 0.5 inch for your unit on the x-axix and 0.2 inch for your unit on the y-axix. To get a good drawing of the curve, choose successive values of x at intervals of halves, beginning with -½. (5 marks).

(ii) From this graph find an approximate maximum value and an exact minimum value for y and the corresponding values of x which make y a maximum or a minimum. (5 marks).

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Look for question 2 at the back of this sheet. P.T.O.

2. 'A' can walk a mile in 2 minutes less time than B would take. In a walking race, 'B' has a start of ¼ mile and A overtakes B in 10 minutes. Assuming both men walk at a uniform rate, find their rates of walking in miles per hour. (20)

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(ii) If $2 \log a - 5 \log b = 3 \log c$, find 'a' in terms of 'b' and 'c'. (4 marks).

(iii) Given $\log_a 4.41 = 2$, calculate the value of 'a'. (4 marks).

(iv) Solve the equation $2^{3-x} = 3^{2x+1}$ giving your answer correct to three decimal places. (6 marks).

4. (i) Write down and simplify an expression for the nth term of the arithmetic progression 3 , 7 , 11 , ... (4 marks).

If the sum of n terms of this progression is $bn + cn^2$ find the values of b and c and the sum of the first thirty terms. (8 marks).

(ii) The product of the first and seventh terms of a geometric progression is equal to the fourth term; andthe sum of the first and fourth terms is 9. Find the sum of the first seven terms of the progression. (8 marks).

5. (i) Draw the graph of $y = (x - 1)(x - 3)^2$ for values of x from -½ to 5, choosing 0.5 inch for your unit on the x-axix and 0.2 inch for your unit on the y-axix. To get a good drawing of the curve, choose successive values of x at intervals of halves, beginning with -½. (5 marks).

(ii) From this graph find an approximate maximum value and an exact minimum value for y and the corresponding values of x which make y a maximum or a minimum. (5 marks).

(iii) By plotting another graph on the same diagram find the roots of the equation $(x - 1)(x - 3)^2 = 5x - 9$. (5 marks).

(iv) From these two graphs find the values of x for which the function $(x - 1)(x - 3)^2$ is always greater than $(5x - 9)$. (5 marks).

---

2. 'A' can walk a mile in 2 minutes less time than B would take. In a walking race, 'B' has a start of ¼ mile and A overtakes B in 10 minutes. Assuming both men walk at a uniform rate, find their rates of walking in miles per hour. (20)

SHAMASH SECONDARY SCHOOL
FINAL EXAMINATION, JUNE, 1965.

Subject : Algebra. Date : 1/6/1965.
Class : 4th year, secondary, sections A & B. Time : 8:00-11:00 a.m.

Attempt all questions :

1. (i) If $m = \frac{2x + y}{x + 2y}$, find an expression for y in terms of m and x.

If also $Y = mx$, find the values of m. (7 marks).

(ii) Resolve into two factors : $c^3 - 27b^3 + a^3 + 9abc$ (7 marks).

(iii) Resolve the expression $5x^2 - 14x + 9$ into two factors and show that the value of this expression is negative when x lies between 1 and 1.8. (6 marks).

3. (i) Compute by logarithms, arranging your work neatly :

$$\sqrt[7]{\frac{(\cos^2 18^\circ\ 47')(\sin^3 48^\circ\ 21')}{(10.09)^3\ (0.000\,2049)}}$$ (6 marks).

(ii) If $2 \log a - 5 \log b = 3 \log c$, find 'a' in terms of 'b' and 'c'. (4 marks).

(iii) Given $\log_a 4.41 = 2$, calculate the value of 'a'. (4 marks).

(iv) Solve the equation $2^{3-x} = 3^{2x+1}$ giving your answer correct to three decimal places. (6 marks).

4. (i) Write down and simplify an expression for the nth term of the arithmetic progression 3 , 7 , 11 , ... (4 marks).

If the sum of n terms of this progression is $bn + cn^2$ find the values of b and c and the sum of the first thirty terms. (8 marks).

(ii) The product of the first and seventh terms of a geometric progression is equal to the fourth term; andthe sum of the first and fourth terms is 9. Find the sum of the first seven terms of the progression. (8 marks).

5. (i) Draw the graph of $y = (x - 1)(x - 3)^2$ for values of x from -½ to 5, choosing 0.5 inch for your unit on the x-axix and 0.2 inch for your unit on the y-axix. To get a good drawing of the curve, choose successive values of x at intervals of halves, beginning with -½. (5 marks).

(ii) From this graph find an approximate maximum value and an exact minimum value for y and the corresponding values of x which make y a maximum or a minimum. (5 marks).

(iii) By plotting another graph on the same diagram find the roots of the equation $(x - 1)(x - 3)^2 = 5x - 9$. (5 marks).

(iv) From these two graphs find the values of x for which the function $(x - 1)(x - 3)^2$ is always greater than $(5x - 9)$. (5 marks).

2. 'A' can walk a mile in 2 minutes less time than B would take. In a walking race, 'B' has a start of ¼ mile and A overtakes B in 10 minutes. Assuming both men walk at a uniform rate, find their rates of walking in miles, per hour. (20)

SHAMASH SECONDARY SCHOOL

4th Quarter Examination, May, 1965.

Subject: Algebra Date: 2/5/1965
Class: 4th Secondary year Time: 8:00-9:30 a.m.

Attempt all questions.

1. (a) Prove that: $(a-a^{-1})(a^{\frac{4}{3}} + a^{-\frac{2}{3}}) = \dfrac{a^2 - a^{-2}}{a^{-\frac{1}{3}}}$ (13 marks)

(b) Evaluate: $\dfrac{x^{\frac{3}{2}} + xy}{xy - y^3} - \dfrac{\sqrt{x}}{\sqrt{x}-y}$ (12 marks)

2. Solve the equation: $\dfrac{6\sqrt{x} - 7}{\sqrt{x} - 1} - 5 = \dfrac{7\sqrt{x} - 26}{7\sqrt{x} - 21}$ (25 marks)

3. Find x from the equation: $3^{2x} = 5^{x+1}$ (25 marks)

4. Compute by logarithms the value of x, arranging your work neatly:

$$\sqrt[7]{\frac{(1.001)^2\ (0.0004061)^{\frac{2}{3}}}{\text{Sin}^3\ 24^\circ\ 21'\ \text{Cos}^2\ 41^\circ\ 57'}}$$ (25 marks)

SHAMASH SECONDARY SCHOOL

4th Quarter Examination, May, 1965.

Subject: Algebra — Date: 2/5/1965
Class: 4th Secondary year — Time: 8:00-9:30 a.m.

Attempt all questions.

1. (a) Prove that: $(a-a^{-1})(a^{\frac{4}{3}} + a^{-\frac{2}{3}}) = \frac{a^2 - a^{-2}}{a^{-\frac{1}{3}}}$ (13 marks)

(b) Evaluate: $\frac{x^{\frac{3}{2}} + xy}{xy - y^3} - \frac{\sqrt{x}}{\sqrt{x}-y}$ (12 marks)

2. Solve the equation: $\frac{6\sqrt{x} - 7}{\sqrt{x} - 1} - 5 = \frac{7\sqrt{x} - 26}{7\sqrt{x} - 21}$ (25 marks)

3. Find x from the equation: $3^{2x} = 5^{x+1}$ (25 marks)

4. Compute by logarithms the value of x, arranging your work neatly:

$$\sqrt[7]{\frac{(1.001)^2\,(0.0004061)^{\frac{3}{3}}}{\sin^3 24^\circ\ 2\overline{1}\ \cos^2 41^\circ\ 5\overline{7}}}$$ (25 marks)

SHAMASH SECONDARY SCHOOL

4th Quarter Examination, May, 1965.

Subject: Algebra — Date: 2/5/1965
Class: 4th Secondary year — Time: 8:00-9:30 a.m.

Attempt all questions.

1. (a) Prove that: $(a-a^{-1})(a^{\frac{4}{3}} + a^{-\frac{2}{3}}) = \dfrac{a^2 - a^{-2}}{a^{-\frac{1}{3}}}$ (13 marks)

(b) Evaluate: $\dfrac{x^{\frac{3}{2}} + xy}{xy - y^3} - \dfrac{\sqrt{x}}{\sqrt{x}-y}$ (12 marks)

2. Solve the equation: $\dfrac{6\sqrt{x} - 7}{\sqrt{x} - 1} - 5 = \dfrac{7\sqrt{x} - 26}{7\sqrt{x} - 21}$ (25 marks)

3. Find x from the equation: $3^{2x} = 5^{x+1}$ (25 marks)

4. Compute by logarithms the value of x, arranging your work neatly:

$$\sqrt[7]{\frac{(1.001)^2\ (0.0004061)^{\frac{2}{3}}}{\text{Sin}^3\ 24^\circ\ 2\overline{1}\ \text{Cos}^2\ 41^\circ\ 5\overline{7}}}$$ (25 marks)

SHAMASH SECONDARY SCHOOL

4th Quarter Examination, May, 1965.

Subject: Algebra — Date: 2/5/1965
Class: 4th Secondary year — Time: 8:00-9:30 a.m.

Attempt all questions.

1. (a) Prove that: $(a-a^{-1})(a^{\frac{4}{3}} + a^{-\frac{2}{3}}) = \dfrac{a^2 - a^{-2}}{a^{-\frac{1}{3}}}$ (13 marks)

(b) Evaluate: $\dfrac{x^{\frac{3}{2}} + xy}{xy - y^3} - \dfrac{\sqrt{x}}{\sqrt{x}-y}$ (13 marks)

2. Solve the equation: $\dfrac{6\sqrt{x} - 7}{\sqrt{x} - 1} - 5 = \dfrac{7\sqrt{x} - 26}{7\sqrt{x} - 21}$ (25 marks)

3. Find x from the equation: $3^{2x} = 5^{x+1}$ (25 marks)

4. Compute by logarithms the value of x, arranging your work neatly:

$$\sqrt[7]{\frac{(1.001)^2\ (0.0004061)^{\frac{2}{3}}}{\sin^3 24^\circ\ 21'\ \cos^2 41^\circ\ 57'}}$$ (25 marks)

Shamash Secondary School
3rd Quarter Examination, March, 1965

Subject: Algebra — Date: 14/3/1965
Class : 4th Year, Section (B) — Time: 10:15 - 11:45 a.m.

Attempt all questions: -

1. Solve simultaneously, the equations:

(i) $x^2 - 2xy + 8y^2 = 3$ ....(1)
$3xy - 2y^2 = 4$ ....(2) (13 marks)

(ii) $(x-2)(y-1) = 3$ ....(1)
$(x+2)(2y-5) = 15$ - (2) (12 marks)

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2. (i) Show that $27 - 8x^3 - 64y^3 - 72xy$ is divisible by $3 - 2(x+2y)$ and find the quotient in this way. (8 marks)

(ii) Resolve into six factors $x^{18} - y^{18}$ (8 marks).

(iii) Find the value of $x^4 + x^2y^2 + y^4$ in terms of a and b, having given: $x + y = 2a$ and $x - y = 2b$ (9 marks)

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3. A man arrives by air at the airport of this city $\frac{3}{4}$ of an hour earlier than the scheduled time, and sets out at once by a taxi, driving to his house at the rate of 20 miles per hour. At the same time, his driver who is supposed to leave his master's house to meet him at the airport in the scheduled time, did so according to plan and, instead, met him on the road to the airport after he has driven his master's private car for a distance of only 50 miles from his house. He immediately picked his master and turned back to his home reaching it exactly 30 minutes earlier than was originally expected. How far is the man's house from the airport and at what rate was his private car driven? (25 marks)

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4. (i) Determine the asymptotes and draw the curve of $y = \frac{x}{x-1}$, for values of x from $x = -2$ to $x = 4$, taking 1 inch as the unit on the x-axis and 0.4 inch as the unit on the y-axis. (7 marks)

(ii) Draw in the same diagram the graph of $y = x(x - 1.5)$ for the same values of x. (7 marks)

ii From your diagram find as accurately as possible

(a) the value of $\frac{1.3}{1.3 - x}$ (5 marks).

(b) two positive numbers differing by 1.5 whose product is 7. (6 marks).

Show in your diagram how each answer is obtained.

Shamash Secondary School
Conditional Examination, Sept.1964

Subject: Algebra
Class: 4th Year Secondary,
Scientific Section.

Date: 2/9/1964
Time: 8:00 - 11:00 a.m.

All questions are to be attempted.

I. (a) The expression x^2+px+q reduces to zero when x equals 2 or 4. Find the values of x for which this expression equals 48.

(10 marks)

(b) The expression $x^4+ax^3+bx^2+cx+12$ is factorable into (x-1), (x+1) and (x+3). Find the values of a, b, c and the other factor.

(10 marks)

II. (a) At what time between eleven and twelve o'clock will the two hands of a watch be at right angle for the second time ?

(10 marks)

(b) A car covers the distance between Mosul and Baghdad in five hours, travelling on the route which lies on the right bank of the river Tigris. Another car travelling at an average speed which is less by 20 kilometres than the first, covers the distance between the two cities which lies on the left bank of the Tigris and which is 50 kilometres longer, in 7½ hours. Find the length of each course.

(10 marks)

III. (a) If $\log_2 (4x-4) = 2$, find the value of $\log_4 x$.

(10 marks)

(b) Compute by logarithms, arranging your work neatly:

$$\sqrt[7]{\frac{(0.1062)^2 \times (0.0071)^3}{(1.005) \times (3.007)^5}}$$

(10 marks)

IV. (a) If a body falls from rest, (neglecting the friction of the air) it will fall 16 ft during the first second, 48 ft during the second second, 80 ft during the third second, 112 ft during the fourth second and so on.
Find the number of seconds it will take a stone to reach the bottom of a well 1936 ft. deep, if it is dropped from the top of the well.

(10 marks)

(b) In a Geometric progression, 1023 times the sum of the first five terms is equal to 31 times the sum of the first ten terms. Find the common ratio.

(10 marks)

(cont'd.p.2)..

Conditional Exam.in Algebra for the 4th Year, Sept.1964. (cont'd.)

V. (a) Sketch the curve of the function (x^4-8x^2) for values of x from -3 to 3 choosing ½ inch as one unit on the x-axis and one tenth of an inch as one unit on the y-axis.

(8 marks)

(b) From this curve find the values of x at which the function (x^4-8x^2) has minimum or maximum values. Find also these minimum and maximum values.

(6 marks)

(c) Plot on the same diagram the graph of y= 3x-10 and from these two graphs find the solution of the equation $x^4-8x^2+10 = 3x$.

(6 marks).

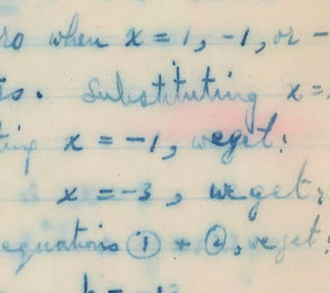
Solutions to Preliminary Exam in Algebra
4th Year, Sept., 1964

1.-(a) Substitute $x=2$ in the expression x^2+px+q + you get $4+2p+q=0$ ... ①
Substitute $x=4$ " " " " " " " $16+4p+q=0$ ... ②
Subtracting equation ① from ②, we get $12+2p=0$ or $p=-6$
∴ from ① $4-12+q=0$ ∴ $q=8$ ∴ $q=8$
∴ when $x^2-6x+8=48$, we get $x^2-6x-40=0$ or $(x-10)(x+4)=0$
∴ $x=10$ or $x=-4$ Ans.

(b) $x^4+ax^3+bx^2+cx+12$ will be equal to zero when $x=1, -1,$ or -3
Since $(x-1)$ and $(x+1)$ and $(x+3)$ are factors. Substituting $x=1$, we get $1+a+b+c+12=0$ --- ①. Substituting $x=-1$, we get:
$1-a+b-c+12=0$ --- ②. " $x=-3$, we get:
$81-27a+9b-3c+12=0$ --- ③. Adding equations ① + ②, we get:
$2+2b+24=0$ or $2b=-26$ or $b=-13$
Dividing eq. ③ by 3, we get $27-9a+3b-c+4=0$ or $27-9a+3(-13)-c+4=0$ or
$-9a+c+8=0$ --- ④. But from ① we get
$a+c=0$ --- ⑤. Subtracting ⑤ from ④ we get
$-8a+8=0$ or $a=-1$. Substituting in ①
we get $1-1-13+c+12=0$ or $c=1$ ∴ $a=-1, b=-13, c=1$ Ans.
Hence the expression is $x^4-x^3-13x^2+x+12$
B.+ $(x-1)(x+1)(x+3)=(x^2-1)(x+3)=x^3+3x^2-x-3$ ∴ the fourth factor =
$(x^4-x^3-13x^2+x+12)\div(x^3+3x^2-x-3)=x-4$ Ans. 2

$$\begin{array}{l|l} x^4-x^3-13x^2+x+12 & x^3+3x^2-x-3 \\ x^4+3x^3-x^2-3x & x-4 \\ \hline -4x^3-12x^2+4x+12 & \\ -4x^3-12x^2+4x+12 & \\ \hline \end{array}$$

I.-(a) Let the time be x minutes after 11, then the hour hand will be pointing $\frac{x}{12}$ divisions after 11. Now are $BC=5-\frac{x}{12}$
∴ $x+\text{arc } BC=45$ or $x+5-\frac{x}{12}=45$
∴ $\frac{11x}{12}=40$ or $11x=480$ or $x=\frac{480}{11}$



or $x=11\times5+\frac{x}{12}-15$
or $x=43\frac{7}{11}$ min. $=43.6\dot{3}$ min $=43$ min $38\frac{2}{11}$ seconds
$x=55\frac{x}{12}-15$
$x=40+\frac{x}{12}$
∴ the time is 43 min $38\frac{2}{11}$ sec. after eleven Ans.

II.(b) Let x kilometres = the distance between the two cities on the right Bank

$\therefore \frac{x}{5}$ k.p.hr. is the average speed of the first car

$\therefore \frac{x+50}{7.5}$ k.p.h. " " " " " " second car $\Big\} \therefore \frac{x}{5} - \frac{x+50}{7.5} = 20$

Multiplying the terms of the equation by 15, we get: $3x - 2(x+50) = 300$ or

$3x - 2x - 100 = 300$ or $x = 400$ Km. Ans.1 = the length of the first Course

$\therefore$ $x+50 = 400+5 = 450$ Km. Ans. 2 = " " " 2nd "

III (a) If $\log_2(4x-4) = 2$, then $\log_4 x = ?$

From the equation we get $4x - 4 = 2^2$ or $4x - 4 = 4$ $\therefore x = 2$

Now $\log_4 x = \log_4 2 = y$ $\therefore 2 = 4^y$ or $2 = 2^{2y}$ $\therefore 2y = 1$ $\therefore y = \frac{1}{2}$

$\therefore \log_4 x = \log_4 2 = \frac{1}{2}$ Ans.

(b) Let $x = \sqrt[7]{\frac{(0.1062)^2 (0.0071)^3}{(1.005)(3.007)^5}}$

$\log 0.1062 = \bar{1}.0261$	$2\log 0.1062 = \bar{2}.0522$	$\log 1.005 = 0.0021$
$\log 0.0071 = \bar{3}.8513$	$3\log 0.0071 = \bar{7}.5539$	$5\log 3.007 = 2.3905$
$\log 1.005 = 0.0021$	$\log$ Num. $= \bar{9}.6061$	$\log$ Den. $= 2.3926$
$\log 3.007 = 0.4781$	$\log$ Den. $= 2.3926$	
	$7\log x = \bar{11}.2135$	
	$\log x = \bar{2}.45907 = \bar{2}.4591$ Correct to 4 dec. pl.	
	$\therefore x = 0.02878$ or $= 2.878 \times 10^{-2}$ Ans.	

IV (a) The distances fallen during the consecutive seconds beginning with the first are: $16 + 48 + 80 + 112 + \ldots$ which is the sum of an A.P. in which $a = 16$, $d = 32$, $S = 1936$, $n = ?$

From the formula $S = \frac{n}{2}\{2a + (n-1)d\}$ we get: $1936 = \frac{n}{2}\{2\times16 + (n-1)\times32\}$ or

$2\times1936 = n\{32 + 32n - 32\}$ or $32n^2 = 3872$ or $n^2 = 121$ or $n = 11$

$\therefore$ the number of seconds which will take the stone to reach the bottom = 11 seconds

Ans.

IV (b) Suppose that the G.P. is $a, ar, ar^2, \ldots$ Then $S_5 = \frac{a(r^5-1)}{r-1}$ and the sixth term $= ar^5$. The sum of the terms from the sixth to the tenth inclusive $= \frac{ar^5(r^5-1)}{r-1}$

$\therefore S_{10} = S_5 + \frac{ar^5(r^5-1)}{r-1}$ or $S_{10} = \frac{a(r^5-1)}{r-1} + \frac{ar^5(r^5-1)}{r-1}$

But $1023\, S_5 = 31\, S_{10}$ or $1023\left[\frac{a(r^5-1)}{r-1}\right] = 31\left[\frac{a(r^5-1)}{r-1} + \frac{ar^5(r^5-1)}{r-1}\right]$ or

$1023\left[\frac{a(r^5-1)}{r-1}\right] = 31\left[\frac{a(r^5-1)}{r-1}\right]\left[1+r^5\right]$ or $1023 = 31(1+r^5)$ or

$1+r^5 = \frac{1023}{31}$ or $1+r^5 = 33$ or $r^5 = 32$ or $r^5 = 2^5$ $\therefore r = 2$ Ans.

x	$f(x)=x^4-8x^2$	$y=3x-10$
-3	9	
-2		
	-16	-4
	9	

[illegible] graph of the function $f(x)=x^4-8x^2$ is the curve shown below

Ans. 1

[illegible] the graph it is clear that:

when $x=-2$, $x^4-8x^2=-16$ minimum value
when $x=0$, $x^4-8x^2=0$ Maximum value
when $x=2$, $x^4-8x^2=-16$ a second minimum value

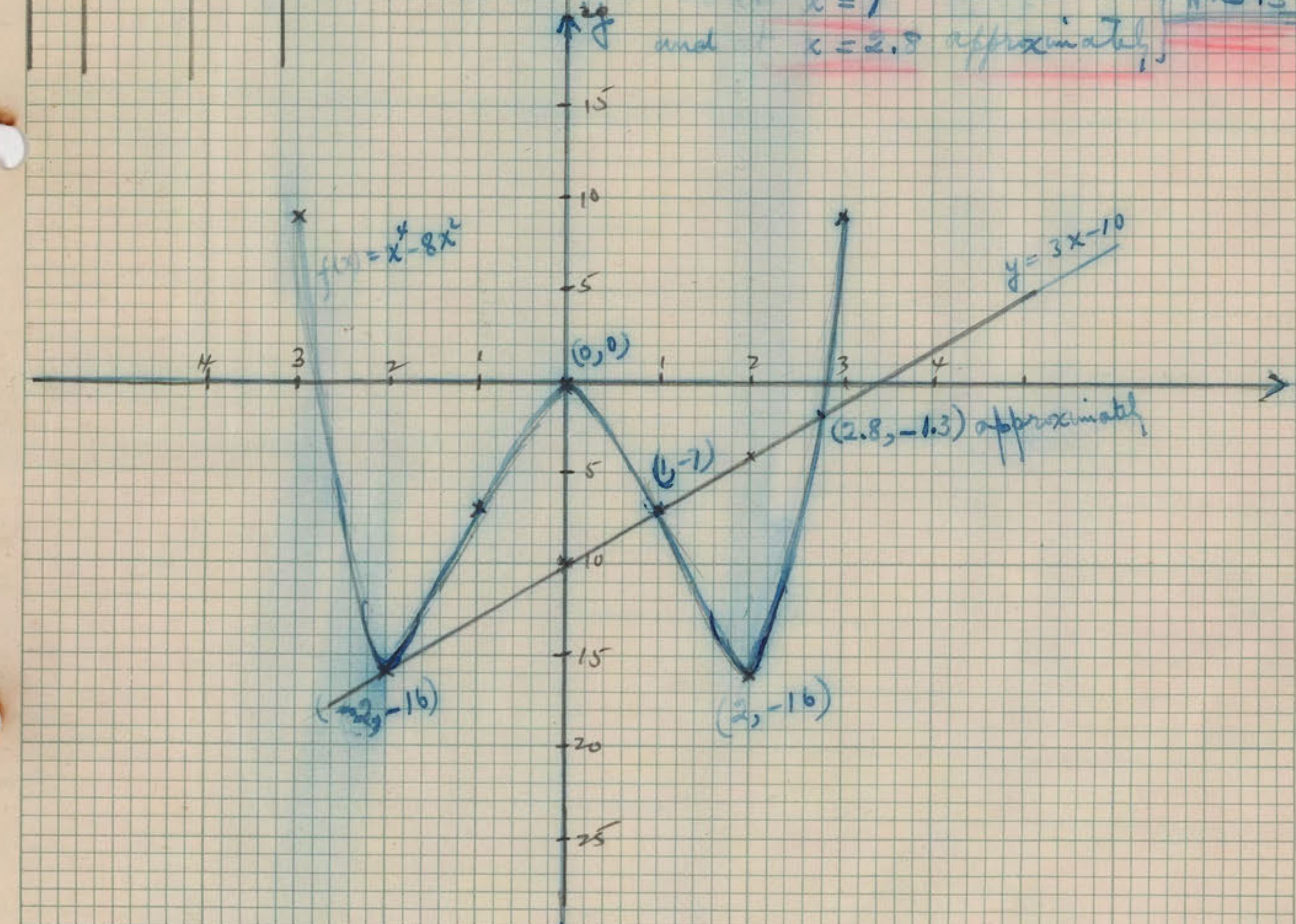
Ans. 2

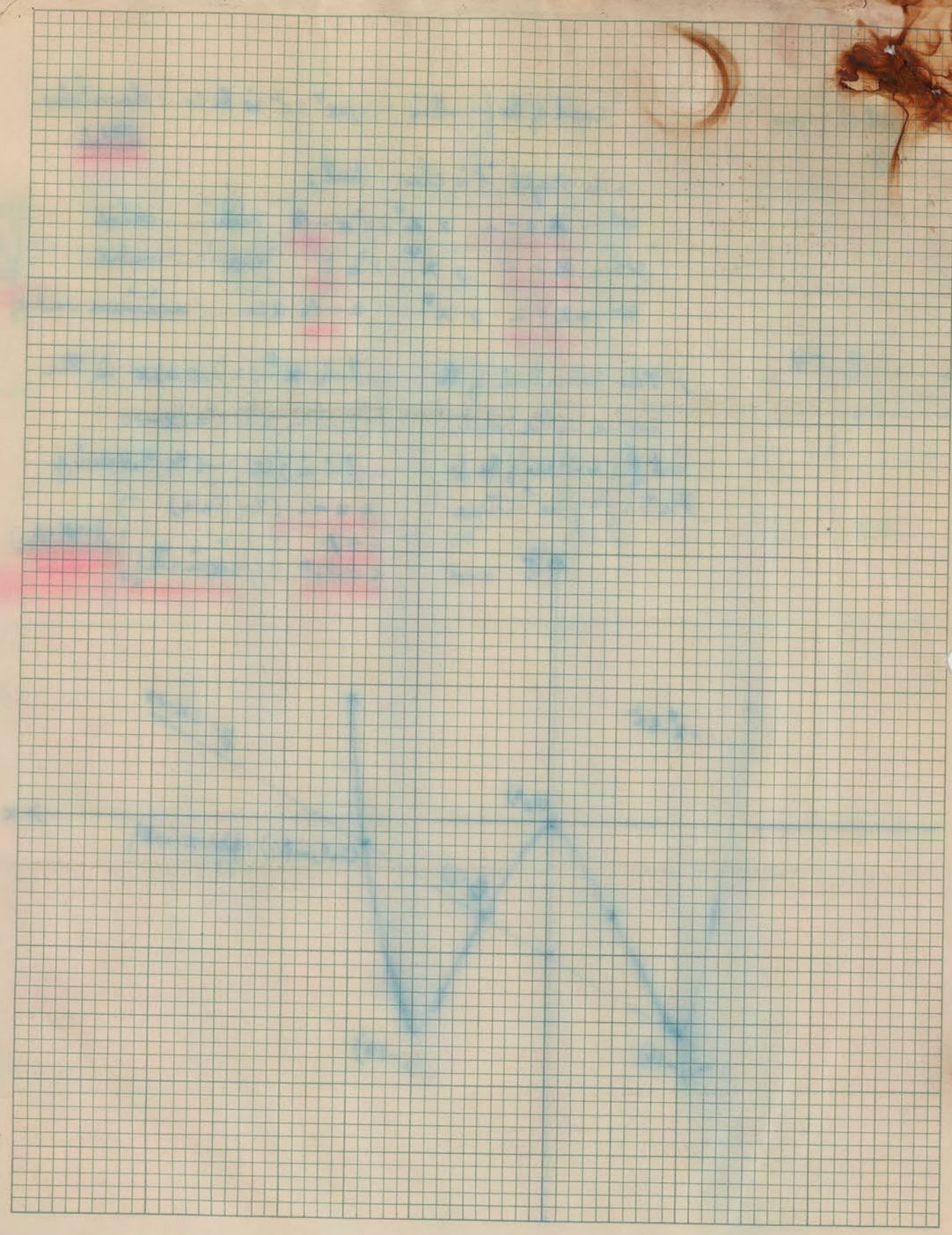
[illegible]) The solution of the equation $x^4-8x^2+10=3x$ is the same as the solution of the equation $x^4-8x^2=3x-10$

But the L.H.S. of the last equation is $f(x)$ and the R.H.S. is y

The two graphs intersect at $x=-2$ (they touch)
at $x=1$
and at $x=2.8$ approximately

Ans. 3





Shamash Secondary School
Conditional Examination, Sept.1964

Subject: Algebra
Class: 4th Year Secondary,
Scientific Section.

Date: 2/9/1964
Time: 8:00 - 11:00 a.m.

All questions are to be attempted.

I. (a) The expression x^2+px+q reduces to zero when x equals 2 or 4. Find the values of x for which this expression equals 48.

(10 marks)

(b) The expression $x^4+ax^3+bx^2+cx+12$ is factorable into (x-1), (x+1) and (x+3). Find the values of a, b, c and the other factor.

(10 marks)

II. (a) At what time between eleven and twelve o'clock will the two hands of a watch be at right angle for the second time ?

(10 marks)

(b) A car covers the distançe between Mosul and Baghdad in five hours, travelling on the route which lies on the right bank of the river Tigris. Another car travelling at an average speed which is less by 20 kilometres than the first, covers the distance between the two cities which lies on the left bank of the Tigris and which is 50 kilometres longer, in 7½ hours. Find the length of each course.

(10 marks)

III. (a) If $\log_2 (4x-4) = 2$, find the value of $\log_4 x$.

(10 marks)

(b) Compute by logarithms, arranging your work neatly:

$$\sqrt[7]{\frac{(0.1062)^2 \times (0.0071)^3}{(1.005) \times (3.007)^5}}$$

(10 marks)

IV. (a) If a body falls from rest, (neglecting the friction of the air) it will fall 16 ft during the first second, 48 ft during the second second, 80 ft during the third second, 112 ft during the fourth second and so on.
Find the number of seconds it will take a stone to reach the bottom of a well 1936 ft. deep, if it is dropped from the top of the well.

(10 marks)

(b) In a Geometric progression, 1023 times the sum of the first five terms is equal to 31 times the sum of the first ten terms. Find the common ratio.

(10 marks)

(cont'd.p.2)..

V. (a) Sketch the curve of the function (x^4-8x^2) for values of x from -3 to 3 choosing ½ inch as one unit on the x-axis and one tenth of an inch as one unit on the y-axis.

(8 marks)

(b) From this curve find the values of x at which the function (x^4-8x^2) has minimum or maximum values. Find also these minimum and maximum values.

(6 marks)

(c) Plot on the same diagram the graph of y= 3x-10 and from these two graphs find the solution of the equation $x^4-8x^2+10 = 3x$.

(6 marks).

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not considered as an exam.

Shamash Secondary School
Final Exams. J u n e, 1 9 6 4.

Paper II

Subject: Algebra — Date: 16/6/1964
Class: 4th Year (Scientific Section) — Time: 8:00-10:00 a.m.

All questions are to be attempted.

I. A and B are two towns 44 miles apart. A cyclist and a motorist travel from A to B. The motorist leaves A 1 hr.36 min. later than the cyclist but they reach B at exactly the same time. If the average speed of the motorist is 18 m.p.h. greater than that of the cyclist, find the average speed of each.

(20 marks)

II. (i) Find the values of x and y if

$$\frac{X}{2}+\frac{Y}{4}=4=\frac{X}{4}+\frac{Y}{2}.$$

(10 marks)

(ii) Find the values of a and b so that $3x^2-4x+1$ can be expressed in the form $a(x-1)^2+b(x-1)$.

(10 marks)

III. (i) The first, second and last terms of an arithmetic progression are x, y and z respectively.

a- Express the number of terms of the progression in terms of x, y and z.

(5 marks)

b- Show that the sum of the progression is

$$\frac{(x+z)(y+z-2x)}{2(y-x)}$$

(5 marks)

(ii) Four positive numbers are in geometric progression. The product of the first and third is 36 and the product of the second and fourth is 324. Find the numbers.

(10 marks)

IV. (i) If S denotes the sum $1+2+3+....+n$, and T denotes the sum $1+2+3+....+(n-1)$, find in terms of n the value of S^2-T^2. Give your answer in its lowest terms.

(10 marks)

(ii) What number must be added to each of the numbers $3, 6, 10\frac{1}{2}$ to form the first three terms of a geometric progression? Find the sum of the first six terms of this progression.

(10 marks)

V.

(cont'd.p.2)

Algebra. 4th Year 16/6/1964

(cont'd.)

V. A ball is thrown vertically upwards into the air from a point which is 20 feet above the sea. After x seconds the height y feet of the ball above the point from which it is thrown is given by $y = 16x(4-x)$.

Draw a graph between x=0 and x=5 showing the relationship between y and x. (Take 1 in. = 1 second and 20 feet respectively). (5 marks)

From the graph, find:

(a) the maximum height above the sea reached by the ball, (5 marks)

(b) how long the ball remains at least 30 feet above the sea, (5 marks)

(c) how many seconds elapse from the time the ball was thrown to the time the ball strikes the sea. (5 marks)

Exam, June 1964
Year Secondary

Let x m.p.h. = average speed of cyclist
$\therefore (x+18)$ m.p.h. = " " " Motorist

$\therefore \frac{44}{x} = \frac{44}{x+18} + 1\frac{36}{60}$ or $\frac{44}{x} = \frac{44}{x+18} + 1\frac{3}{5}$ or $\frac{44}{x} = \frac{44}{x+18} + \frac{8}{5}$

$\therefore 5\times 44(x+18) = 5\times 44x + 8x(x+18)$ or $220x + 3960 = 220x + 8x^2 + 144x$

or $8x^2 + 144x - 3960 = 0$ or $x^2 + 18x - 495 = 0$ or

$(x+33)(x-15) = 0$ $\therefore x = -33$ (to be discarded) + $x = 15$ m.p.h. Ans.
= av. sp. of cyclist

$\therefore x + 18 = 15 + 18 = 33$ m.p.h. Ans.
= av. sp. of motorist

II. (i) $\frac{x}{2} + \frac{y}{4} = 4 = \frac{x}{4} + \frac{y}{2}$ $\therefore \frac{x}{2} + \frac{y}{4} = 4$ .... ①

$\frac{x}{4} + \frac{y}{2} = 4$ .... ②

$\therefore 2x + y = 16$ .... ①a $\therefore \{4x + 2y = 32$ .... ③$\}$

$x + 2y = 16$ .... ②a $\{x + 2y = 16$ .... ②$\}$

$\therefore 3x = 16$ $\therefore x = \frac{16}{3}$ from ②, $y = 16 - 2x = 16 - \frac{32}{3} = \frac{16}{3}$

$\therefore x = \frac{16}{3}$
$y = \frac{16}{3}$ Ans.

(ii) In order that the expression $3x^2 - 4x + 1$ be expressed in the form $a(x-1)^2 + b(x-1)$, the two forms should be identical. Hence the equation: $3x^2 - 4x + 1 = a(x-1)^2 + b(x-1)$ is an identity which is true for all values of x. Now let $x = 0$, then $1 = a(-1)^2 + b(-1)$ or

$a - b = 1$ .... ①. Also let $x = 2$, then $5 = a + b$ or
$a + b = 5$ .... ②

$\therefore 2a = 6$ $\therefore a = 3$ and $b = 2$ $\therefore \{a = 3, b = 2\}$ Ans.

III. (i) (a) Let the No. of terms be n. Now the 1st term $= x$ and the common difference $d = y - x$ $\therefore$ the n^{th} term $z = x + (n-1)(y-x)$

$\therefore (n-1)(y-x) = z - x$ $\therefore n - 1 = \frac{z-x}{y-x}$ $\therefore n = \frac{z-x}{y-x} + 1$

or $n = \frac{z + y - 2x}{y - x}$ Ans.

(i) (b) Sum of the Progression $S_n = \frac{n}{2}(a+l_n) = \frac{z+y-2x}{2(y-x)}[x+z]$

or $S_n = \frac{(x+z)(y+z-2x)}{2(y-x)}$ Ans.

(ii) Let the four terms be a, ar, ar^2, ar^3. Then:

(a) $(ar^2) = 36$ or $a^2r^2 = 36$ ---① } Dividing ② by ①, we get

and $(ar)(ar^3) = 324$ or $a^2r^4 = 324$ ---② }

$r^2 = \frac{324}{36} = 9$ $\therefore r^2 = 9$ $\therefore r = 3$ (the negative root is to be excluded)

from ① $a^2r^2 = 36$ $\therefore 9a^2 = 36$ $\therefore a^2 = 4$ $\therefore a = 2$. Therefore the four positive numbers are: 2, 6, 18, 54 Ans.

V (i) $S = 1+2+3+\cdots+n$ $\therefore S = \frac{n}{2}(1+n) = \frac{n(n+1)}{2}$

$T = 1+2+3+\cdots+(n-1)$ $\therefore T = \frac{n-1}{2}[1+(n-1)] = \frac{n(n-1)}{2}$

but $S^2 - T^2 = (S+T)(S-T) = \left[\frac{n(n+1)}{2} + \frac{n(n-1)}{2}\right]\left[\frac{n(n+1)}{2} - \frac{n(n-1)}{2}\right]$

$\therefore S^2 - T^2 = \frac{n^2+n+n^2-n}{2} \times \frac{n^2+n-n^2+n}{2} = n^2 * n = n^3$ Ans.

(ii) 3, 6, $10\frac{1}{2}$ Let x be the number to be added to each of the three numbers to make a G.P. out of them. Then the three terms are $(3+x), (6+x), (\frac{21}{2}+x)$. Hence $r = \frac{\frac{21}{2}+x}{6+x} = \frac{6+x}{3+x}$

multiplying across, we get: $(6+x)^2 = (3+x)(\frac{21}{2}+x)$ or

$36+12x+x^2 = \frac{(3+x)(21+2x)}{2}$ or $36+12x+x^2 = \frac{63+27x+2x^2}{2}$ or

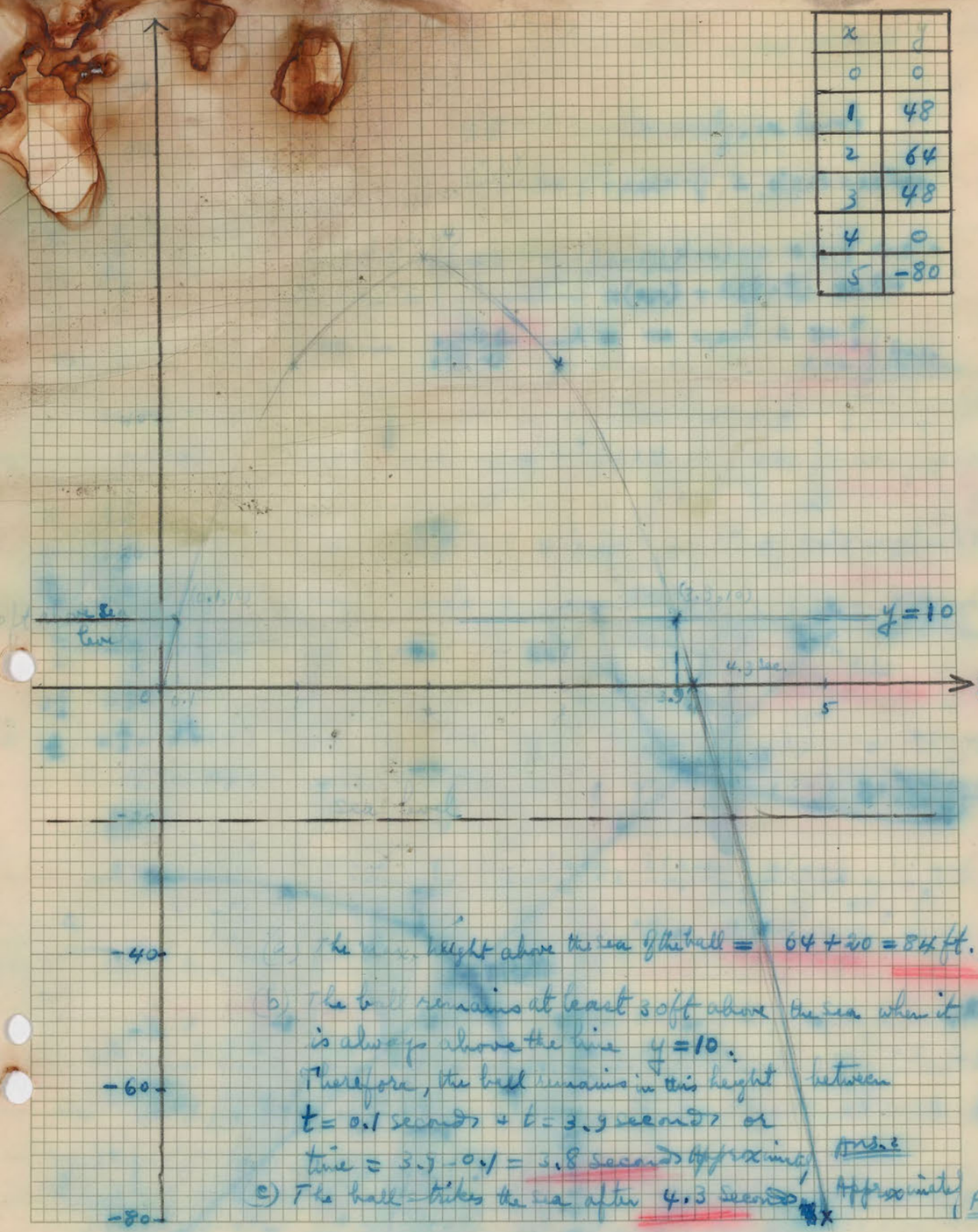
$72+24x+2x^2 = 63+27x+2x^2$ or $3x = 9$ $\therefore x = 3$ Ans.

$\therefore$ the numbers are 6, 9, $\frac{21}{2}$, ...

$S_6 = \frac{a(r^n-1)}{r-1} = \frac{6\left[\left(\frac{3}{2}\right)^6 - 1\right]}{\frac{3}{2}-1} = \frac{6\left[\frac{729}{64}-1\right]}{\frac{1}{2}} = 12\left[\frac{665}{64}\right] = \frac{1995}{16} =$

or $\frac{11}{16}$

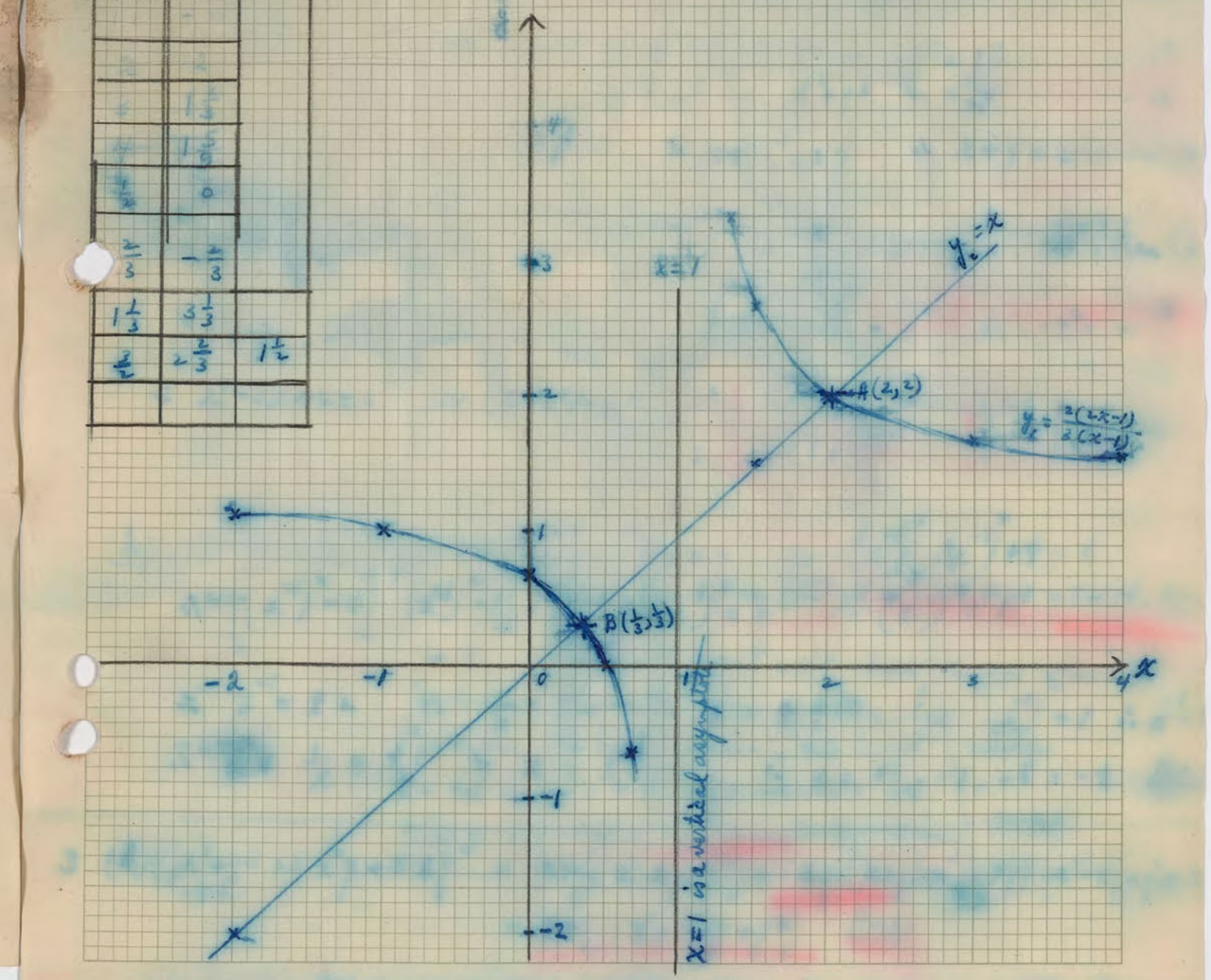
x	y
0	0
1	48
2	64
3	48
4	0
5	-80



b) The ball remains at least 30 ft above the sea when it is always above the line $y = 10$.
Therefore, the ball remains in this height between
$t = 0.1$ seconds & $t = 3.9$ seconds or
time $= 3.9 - 0.1 = 3.8$ seconds approximately. Ans.

c) The ball strikes the sea after 4.3 seconds approximately. Ans.

	$1\frac{1}{3}$	
	$1\frac{2}{3}$	
	2	
	$2\frac{1}{3}$	
	$2\frac{2}{3}$	
	3	
	$3\frac{1}{3}$	
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$
$\frac{2}{3}$	0	
$\frac{5}{6}$	$-\frac{2}{3}$	
$1\frac{1}{3}$	$3\frac{1}{3}$	
$\frac{3}{2}$	$2\frac{2}{3}$	$1\frac{1}{2}$



(b) } graphs

$x^3+y^3=9$ ..... ① ; $x^2y+xy^2=6$ ..... ② ; multiplying equation ② by 3 + adding to equation ①, we get: $3x^2y+3xy^2=18$ ..... ②

$x^3+y^3=9$ ..... ①

$\therefore x^3+3x^2y+3xy^2+y^3=27 \quad \therefore (x+y)^3=27 \quad \therefore x+y=3$ ..... ③

from ② $xy(x+y)=6 \quad \therefore 3xy=6 \quad \therefore xy=2$ ..... ④ . From ③, $y=3-x$, substituting in ④, we get $x(3-x)=2 \quad \therefore 3x-x^2=2$

$\therefore x^2-3x+2=0 \quad$ or $(x-2)(x-1)=0 \quad \therefore$ $x=2$, $y=1$ } Ans. 1 and $x=1$, $y=2$ } Ans. 2

(b) $x^{-4}+4=5x^{-2} \quad$ or $x^{-4}-5x^{-2}+4=0 \quad$ or $(x^{-1})^4-5(x^{-1})^2+4=0$

$\therefore [(x^{-1})^2-4][(x^{-1})^2-1]=0 \quad \therefore (x^{-1})^2=4 \quad$ or $(x^{-1})^2=1$

$\therefore x^{-1}=\pm 2 \quad$ or $\frac{1}{x}=\pm 2 \quad \therefore x=\pm\frac{1}{2} \quad$ also $(x^{-1})^2=1 \quad \therefore x^{-1}=\pm 1$

$\therefore \frac{1}{x}=\pm 1 \quad \therefore x=\pm 1 \quad \therefore x=+1, -1, +\frac{1}{2}, -\frac{1}{2}$ Ans.

3 (a) (i) $x^3+y^3+5x^2y+5xy^2 = (x+y)(x^2-xy+y^2)+5xy(x+y) = (x+y)[x^2-xy+y^2+5xy]$

$= (x+y)(x^2+4xy+y^2)$ Ans.

(ii) $(x+y+z)^2+x^2-y^2-z^2 = x^2+y^2+z^2+2xy+2xz+2yz+x^2-y^2-z^2$

$= 2x^2+2xy+2xz+2yz = 2[x^2+xy+xz+yz]$

$= 2[x(x+y)+z(x+y)] = 2(x+y)(x+z)$ Ans.

(iii) $(a^2-b^2)(x^2-y^2)+4abxy = a^2x^2-a^2y^2-b^2x^2+b^2y^2+4abxy$

$= a^2x^2+2abxy+b^2y^2-(b^2x^2-2abxy+a^2y^2)$

$= (ax+by)^2-(bx-ay)^2 = (ax+by+bx-ay)(ax+by-bx+ay)$

$= [(a+b)x+(b-a)y][(a-b)x+(b+a)y]$ Ans.

3 (b) $x + \frac{1}{x} = \sqrt{3}$ Prove that $x^3 + \frac{1}{x^3} = 0$

$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right) = \left(x + \frac{1}{x}\right)\left(x^2 + 2 + \frac{1}{x^2} - 3\right) = \left(x + \frac{1}{x}\right)\left[\left(x + \frac{1}{x}\right)^2 - 3\right]$

$= \sqrt{3}\left[(\sqrt{3})^2 - 3\right] = \sqrt{3}(3 - 3) =$ zero Q.E.D.

4 (a) Let the length of the course be = x yards

∴ No. of seconds taken by the first boat = $\frac{x}{4}$ seconds = t_1

also " " " " " " 2nd " = $\frac{\frac{x}{2}}{3\frac{1}{2}} + \frac{\frac{x}{2}}{4\frac{1}{2}} = \frac{x}{7} + \frac{x}{9} = \frac{16x}{63}$ seconds = t_2

∴ $t_1 = \frac{x}{4} = \frac{63x}{4 \times 63}$ seconds

∴ $t_2 = \frac{4 \times 16x}{4 \times 63} = \frac{64x}{4 \times 63}$ "

} ∴ the first boat wins the race by $\frac{64x - 63x}{4 \times 63} = \frac{x}{4 \times 63}$ seconds Ans.

(b) Let the distance of riding be = x miles

$\frac{x}{b}$ hours = time taken in riding

$\frac{x}{c}$ hours = time " " walking

} ∴ $\frac{x}{b} + \frac{x}{c} = a$ ∴ $\frac{cx + bx}{bc} = a$

or $x(b + c) = a \cdot b \cdot c$ ∴ $x = \frac{a \cdot b \cdot c}{b + c}$ miles Ans.

(i) It is required to find the sum of $2 + 4 + 6 + \ldots$ to $(n+1)$ terms

∴ $S = \frac{n}{2}\{2a + (n-1)d\}$ or $S = \frac{n+1}{2}\{2a + (n+1-1)(2)\}$ = ~~$n + 2n + 2(n+1)$~~

$S = \frac{n+1}{2}\{4 + 2n\} = (n+1)(n+2) = n^2 + 3n + 2$ Ans.

(ii) $S = 22 + 33 + 44 + \ldots$ to ten terms

$S = \frac{10}{2}\{2 \times 22 + (10-1) \times 11\} = 5(44 + 99) = 5 \times 143 = 715$ Ans.

(iii) the distances fallen in the different seconds are: 16, 48, 80, ...

$t_{12} = a + (n-1)d = 16 + 11 \times 32 = 16 + 352 = 368$ ft. Ans. 1

$S = \frac{n}{2}\{a + l\} = \frac{12}{2}(16 + 368) = 6 \times 384 = 2304$ ft. Ans. 2

(iv) Total distance covered = $10 + 10 + 5 + 2\frac{1}{2} + 1\frac{1}{4} + \ldots$ to infinity

~~$= 10 + \left[\frac{10}{1 - \frac{1}{2}}\right]$~~ $= 10 + \left(\frac{10}{1 - \frac{1}{2}}\right) = 10 + 20 = 30$ Ans.

6.(a) $$x = \sqrt[5]{\frac{(1.005)^3 \times (0.0004007)^5}{(0.06109)^2 \times (10.71)^{\frac{1}{3}}}}$$

$\log 1.005 = 0.0021$ ✓	$3\log 1.005 = 0.0063$	$2\log 0.06109 = \bar{2}.5718$ ✓	
$\log 0.0004007 = \bar{4}.6029$ ✓	$5\log 0.0004007 = \bar{17}.0145$	$\frac{1}{3}\log 10.71 = 0.3433$	
$\log 0.06109 = \bar{2}.7859$ ✓	$\log$ Num. $= \bar{17}.0208$	$\log$ Den. $= \bar{3}.9151$ ✓	③
$\log 10.71 = 1.0298$ ✓	$\log$ Den. $= \bar{3}.9151$ ③		
	$5\log x = \bar{15}.1057$ ✓		
	$\log x = \bar{3}.02114$	②	
	$x = 1.050 \times 10^{-3} = 0.001050$		

Ans.

(b) $\log_{10} 2 = 0.30103$, $\log_{10} 3 = 0.47712$, $\log_3 648 = ?$

$$\log_3 648 = \log_3 2^3 \times 3^4 = \log_3 2^3 + \log_3 3^4 = 3\log_3 2 + 4\log_3 3 =$$

$$= 4 + 3\left(\frac{\log_{10} 2}{\log_{10} 3}\right) = 4 + 3 \times \frac{0.30103}{0.47712} = 4 + \frac{.90309}{.47712}$$

$= 4 + 1.892794 = 5.892794 = 5.89279$ Correct to five decimal places

Ans.

② ⑥

$3^7 = 648$

$3\log 2$: $3 \times 0.30103 = .90309$

$4\log 3$: $4 \times 0.47712 = 1.90848$

2.811157

$$\log_3 648 = \frac{\log_{10} 648}{\log_{10} 3} = \frac{2.811157}{0.47712} = 5.892794 = 5.89279$$ Correct to 5 dec. pl.

Ans

Shamash Secondary School
Final Exams. June, 1964.

Subject: Algebra — Date: 3/6/1964
Class: 4th Secondary (Scientific Section) — Time: 8:00-10:45 a.m.

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1. (a) Draw the graph of $Y = \frac{2(2X-1)}{3(X-1)}$ for values of X from X=-2 to X =4, choosing one inch to represent one unit on the X-axis and 9 tenths of an inch to represent one unit on the Y-axis. (8 marks)

   (b) Plot another graph on the same axes to find a value of X for which $\frac{2(2X-1)}{3(X-1)} = X$ and verify the result by solving this equation algebraically. (8 marks)

2. (a) Solve simultaneously:

   $X^3+Y^3 = 9$
   $X^2Y+XY^2=6$ (8 marks)

   (b) Find all values of X which satisfy the equation:

   $X^{-4} + 4 = 5X^{-2}$ (8 marks)

3. (a) Find the factors of:

   i- $X^3+ Y^3+ 5X^2Y + 5XY^2$ (4 marks)

   ii- $(X+Y+Z)^3 + X^2-Y^2-Z^2$ (4 marks)

   iii- $(a^2-b^2)(X^2-Y^2)+ 4abxy$ (4 marks)

   (b) If $X+\frac{1}{X} = \sqrt{3}$, prove that $X^3+ \frac{1}{X^3} = 0$ (8 marks)

4. (a) One boat in a race was rowed over the course at an average pace of 4 yards a second; the other moved over the first half of the course at the rate of 3½ yards a second, and over the last half at the rate of 4½ yards a second. Which of them won and by how many seconds ? (8 marks)

   (b) A person has 'a' hours free. How far can he ride at 'b' miles an hour so that walking back at 'c' miles an hour he may reach home in time ? (8 marks)

(cont'd.p.2)

(cont'd.)

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5. (i) What is the sum of the first (n+1) even numbers ?

(4 marks)

(ii) What is the sum of the first ten numbers beginning with 22 that are divisible by 11 ?

(4 marks)

(iii) A body falling freely falls approximately 16 ft., in the first second, and in each succeeding second 32 ft. more than in the second immediately preceding. If a stone dropped from a stationary balloon reaches the ground in 12 seconds, how far does it fall in the last second ? How high is the balloon ?

(4 marks)

(iv) If it were possible for a rubber ball to fall 10 ft., and bound back 5 ft., then to fall 5 ft., and bound back 2½ ft., and to continue this forever, what is the limit of the total distance through which the ball would pass ?

(4 marks)

6. (a) Compute by logarithms the value of the following, arranging your work neatly:

$$\sqrt[5]{\frac{(1.005)^3 \times (0.000407)^5}{(0.06109)^2 \times (10.71)^{\frac{1}{3}}}}$$

(8 marks)

(b) Find, correct to <u>five</u> decimal places, the value of $\log_3 648$, having given:

$$\log_{10} 2 = 0.30103 \quad \text{and} \quad \log_{10} 3 = 0.47712$$

(8 marks)

Shamash Secondary School
3rd Quarter Examination, March 1964

Subject: Algebra — Date: 26/3/1964
Class: 4th Year Secondary — Time: 10:15-11.45

Attempt all questions:

1. An education Committee spent £P in one year awarding n scholarships at Secondary schools. The school fees of the scholars, amounting to £F, were paid in each case; and in addition some of the scholars received a grant of £a and the remainder a grant of £b. How many received the grant of £a ?

(20 marks)

2. Solve each of the following equations:-

(i) $(X^2+2)^2 + 198 = 29(X^2+2)$ (10 marks)

(ii) $X^3+7X^2+7X-15 = 0$ (10 marks)

3. Solve the two simultaneous equations:-

$X^2+4Y^2+80 = 15X+30Y$ (20 marks)

$XY = 6$

4. At what time between ten and eleven O'clock are the hands of a watch at right angles for the second time.

(20 marks)

5. Find the value of $X^4-47X^2Y^2+Y^4$ in terms of p and q when $X+Y = p$ and $X-Y = q$.

(20 marks)

Shamash Secondary School
Conditional Exa mination, Sept.63

Subject: Algebra — Date: 12/9/1963
Class: 4th Year Secondary — Time: 8.30-11.00 a.m.

Attempt all q uestions:

1. (i) The equation $\frac{7X + 2}{X^2 - 4} = \frac{A}{X-2} + \frac{B}{X+2}$ is true for all values of X.

Find the values of A & B. (8 marks)

(ii) The expression $X^3 + pX^2 + qX + 6$ is factorable into (X-1) and (X+2). Find the values of p and q and find the third factor.

(8 marks)

2. Solve for X the following equations, rejecting all extraneous roots:

(i) $\frac{1}{1 - X} + \frac{1}{\sqrt{X} + 1} + \frac{1}{\sqrt{X} - 1} = 0$ (6 marks)

(ii) $(X-7)^{\frac{1}{2}} = \sqrt{X - 7}$ (6 marks)

(iii) $3X^{-\frac{2}{3}} - 10X^{-\frac{1}{3}} + 3 = 0$ (6 marks)

3. (i) Solve for X, using tables if necessary:

$20^{x} = 2^{x+2}$

(ii) Compute by logarithms:

$$\sqrt[5]{\frac{(0.0012)^2 \times (1.003)^3}{7515 \times 2.004}}$$

(8 marks)

4. A man travels 108 miles, and finds that he could have made the journey in $4\frac{1}{2}$ hours less, had he travelled 2 miles an hour faster. At what rate did he travel ?

(16 marks)

5. (i) Find by series the value of the recurring fraction $0.\dot{3}\dot{2}\dot{0}\dot{5}$

(8 marks)

(ii) The three digits of a number are in arithmetical progression. The number itself divided by the sum of the digits is 48. The number formed by the same digits in reverse order is 396 less than the original number. What is the number ?

(8 marks)

Shamash Secondary School
Cond.Exam. September, 1963.

Algebra
4th Year Secondary.

12/9/63

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6. Draw the graph of $Y = \frac{1}{2}X^3$ for values of X between -3 and 4 taking $\frac{1}{2}$ inch to represent one unit on the X-axis and two tenths of an inch to represent one unit on the Y-axis. By drawing other graphs on the same figure, solve the equations:

(i) $\frac{1}{2}X^3 - \frac{7}{2}X - 3 = 0$ (6 marks)

(ii) $\frac{1}{2}X^3 + \frac{3}{2}X - 2 = 0$ (6 marks)

(iii) From the graph find the range of values of X for which $\frac{1}{2}X^3$ is greater than $\frac{7}{2}X + 3$.

(6 marks).

Shamash Secondary School
Conditional Exa mination, Sept.63

Subject: Algebra
Class: 4th Year Secondary

Date: 12/9/1963
Time: 8.30-11.00 a.m.

Attempt all q uestions:

1. (i) The equation $\frac{7X + 2}{X^2 - 4} = \frac{A}{X-2} + \frac{B}{X+2}$ is true for all values of X.

Find the values of A & B. (8 marks)

(ii) The expression $X^3 + pX^2 + qX + 6$ is factorable into (X-1) and (X+2). Find the values of p and q and find the third factor.

(8 marks)

2. Solve for X the following equations, rejecting all extraneous roots:

(i) $\frac{1}{1 - X} + \frac{1}{\sqrt{X} + 1} + \frac{1}{\sqrt{X} - 1} = 0$ (6 marks)

(ii) $(X-7)^{\frac{1}{2}} = \sqrt{X - 7}$ (6 marks)

(iii) $3X^{-\frac{2}{3}} - 10X^{-\frac{1}{3}} + 3 = 0$ (6 marks)

3. (i) Solve for X, using tables if necessary:

$20^{x} = 2^{x+2}$

(ii) Compute by logarithms:

$$\sqrt[5]{\frac{(0.0012)^2 \times (1.003)^3}{7515 \times 2.004}}$$

(8 marks)

4. A man travels 108 miles, and finds that he could have made the journey in $4\frac{1}{2}$ hours less, had he travelled 2 miles an hour faster. At what rate did he travel ?

(16 marks)

5. (i) Find by series the value of the recurring fraction $0.\dot{3}2\dot{0}5$

(8 marks)

(ii) The three digits of a number are in arithmetical progression. The number itself divided by the sum of the digits is 48. The number formed by the same digits in reverse order is 396 less than the original number. What is the number ?

(8 marks)

Shamash Secondary School
Cond.Exam. September, 1963.

Algebra
4th Year Secondary.

12/9/63

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6. Draw the graph of $Y = \frac{1}{2}X^3$ for values of X between -3 and 4 taking $\frac{1}{2}$ inch to represent one unit on the X-axis and two tenths of an inch to represent one unit on the Y-axis. By drawing other graphs on the same figure, solve the equations:

(i) $\frac{1}{2}X^3 - \frac{7}{2}X - 3 = 0$ (6 marks)

(ii) $\frac{1}{2}X^3 + \frac{3}{2}X - 2 = 0$ (6 marks)

(iii) From the graph find the range of values of X for which $\frac{1}{2}X^3$ is greater than $\frac{7}{2}X + 3$.

(6 marks).

Shamash Secondary School
Final Exams. June, 1963.

Subject: Algebra
Class: 4th Year Secondary

Date: 9/6/1963
Time: 8:00-10.30 a.m.

Attempt all questions.

1. (a) Resolve into three factors: $2X^3 - 9X^2 + 7X + 6$. (10 marks)

(b) Resolve into four factors: $4(Xy + mn)^2 - (X^2 + y^2 - m^2 - n^2)^2$. (10 marks)

2. (a) Given $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{6} = 2.440$,
Find to two places of decimals the value of:
$(3 - \sqrt{2})(7 + 4\sqrt{3}) \div (2\sqrt{3} - 3)$,
rationalising the denominator first. (10 marks)

(b) Solve for X :

$$\frac{2\sqrt{X} - 1}{2\sqrt{X} + \frac{4}{3}} = \frac{\sqrt{X} - 2}{\sqrt{X} - \frac{4}{3}}$$ (10 marks)

3. (a) Solve for X without using the tables:

$$(\sqrt{3}\sqrt{2})^X = 36$$ (10 marks)

(b) Compute by logarithms, arranging your work neatly:

$$\sqrt[7]{\frac{(0.0002003)^{\frac{2}{3}} (0.04031)^{\frac{3}{5}}}{1.004 \times 9.006}}$$ (10 marks)

(cont'd.p.2)

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4. (a) Show that the sum of n terms of the series

$1 + \frac{1}{3} + \frac{1}{9} + \ldots\ldots$ is $1.5 - \frac{1}{2X3^{n-1}}$

How many terms of this series must be taken to make the sum equal to $\left(\frac{3}{2} - \frac{1}{13122}\right)$? (10 marks)

(b) The first, second and fourth terms of an Arithmetical progression themselves form three successive terms of a Geometrical progression.
Show that, if the common difference is not zero, it is equal to the first term.
(10 marks).

5. Plot the graph of the function $X^2 - 6X + 5$ for values of X from -1 to 7 choosing $\frac{1}{2}$ inch as one unit on the axis of X, and three tenths of an inch as one unit on the axis of y.
From your graph, find:

(a) the least value of the function.

(b) the roots of the equation $X^2 + 5 = 6X$

(c) by drawing another graph on the same figure, find the values of X between which the given function is less than (X-1).

(d) find from the resulting figure the roots of the equation
$X^2 - 6X + 5 = X-1$ (20 marks)

Year Sunday, Shamash School

$2x^3-9x^2+7x+6$ By trial & error when $x=2$, then

$2x^3-9x^2+7x+6 = 2x2^3-3x2^2+7x2+6 = 16-36+14+6=0 \therefore (x-2)$ is a fac

$\therefore 2x^3-9x^2+7x+6 = 2x^2(x-2)-5x(x-2)-3(x-2) = (x-2)(2x^2-5x-3)$

$\therefore 2x^3-9x^2+7x+6 = (x-2)(2x+1)(x-3)$ Ans.

(b) $4(xy+mn)^2-(x^2+y^2-m^2-n^2)^2 = [2(xy+mn)+(x^2+y^2-m^2-n^2)][2(xy+mn)-(x^2+y^2-m^2-n^2)]$

$= (2xy+2mn+x^2+y^2-m^2-n^2)(2xy+2mn-x^2-y^2+m^2+n^2)$

$= [(x+y)^2-(m-n)^2][(m+n)^2-(x-y)^2] = (x+y+m-n)(x+y-m+n)(m+n+x-y)(m+n-x+y)$

Ans.

2 (a) $\sqrt{2}=1.414$, $\sqrt{3}=1.732$, $\sqrt{6}=2.449$, $\frac{(3-\sqrt{2})(7+4\sqrt{3})}{2\sqrt{3}-3} = \frac{(3-\sqrt{2})(7+4\sqrt{3})(2\sqrt{3}+3)}{(2\sqrt{3}-3)(2\sqrt{3}+3)}$

$= \frac{(21+12\sqrt{3}-7\sqrt{2}-4\sqrt{6})(2\sqrt{3}+3)}{12-9} = \frac{42\sqrt{3}+72-14\sqrt{2}-24\sqrt{2}+63+36\sqrt{3}-21\sqrt{2}-12}{3}$

$= \frac{135+78\sqrt{3}-45\sqrt{2}-26\sqrt{6}}{3} = 45+26\sqrt{3}-15\sqrt{2}-\frac{26}{3}\sqrt{6}$

$= 45+26\times1.732-15\times1.414-\frac{26\times2.449}{3} = 45+45.032-21.210-21.1466 = 90.032-42.3566 = 47.6754$

$= 47.68$ Correct to 2 dec. places Ans.

(b) $\frac{2\sqrt{x}-1}{2\sqrt{x}+\frac{4}{3}} = \frac{\sqrt{x}-2}{\sqrt{x}-\frac{4}{3}}$ or $\frac{2\sqrt{x}-1}{6\sqrt{x}+4} = \frac{\sqrt{x}-2}{3\sqrt{x}-4}$. Multiplying across, we have;

$(6\sqrt{x}+4)(\sqrt{x}-2) = (2\sqrt{x}-1)(3\sqrt{x}-4)$ or $6x-8\sqrt{x}-8 = 6x-11\sqrt{x}+4$

$\therefore 3\sqrt{x}=12 \therefore \sqrt{x}=4 \therefore x=16$ Ans.

3 (a) $(\sqrt{3}\sqrt{2})^x = 36 \therefore (3^{\frac{1}{2}}\times2^{\frac{1}{2}})^x = 36 \therefore (3\times2)^{\frac{x}{2}} = 6^2$ or $6^{\frac{x}{2}} = 6^2 \therefore \frac{x}{2}=2 \therefore x=4$ Ans.

(b) Let $x = \sqrt[7]{\frac{(0.0002003)^{\frac{2}{3}}\times(0.04031)^{\frac{3}{5}}}{1.004\times9.006}}$

$\log 0.0002003 = \bar{4}.3016$	$\frac{2}{3}\log 0.0002003 = \bar{3}.53440$	$\log 1.004 = 0.0017$	
$\log 0.04031 = \bar{2}.6054$	$\frac{3}{5}\log 0.04031 = \bar{1}.16324$	$\log 9.006 = 0.9545$	
$\log 1.004 = 0.0017$	$\log$ Num. $= 4.69764$	$\pm$	
$\log 9.006 = 0.9545$	$\log$ Den. $= 0.95620$	$\log$ Den $= 0.9562$	
$2\log 0.0002003 = \bar{8}.6032$	$7\log x = \bar{5}.74144$		
$3\log 0.04031 = \bar{5}.8162$	$\log x = \bar{1}.391634 = \bar{1}.3916$		
	$x = 0.2463$ Ans.		

4(a) $1 + \frac{1}{3} + \frac{1}{9} + \cdots$ to n terms $\quad S_n = \frac{1\left[1-\left(\frac{1}{3}\right)^n\right]}{1-\frac{1}{3}} = \frac{3}{2}\left(1-\frac{1}{3^n}\right) = \frac{3}{2} - \frac{1}{2\times 3^{n-1}}$

$\therefore S_n = 1.5 - \frac{1}{2\times 3^{n-1}}$ Ans.1

When $S_n = \frac{3}{2} - \frac{1}{13122}$, then $1.5 - \frac{1}{2\times 3^{n-1}} = \frac{3}{2} - \frac{1}{13122}$

$\therefore \frac{1}{2\times 3^{n-1}} = \frac{1}{13122} \quad \therefore 2\times 3^{n-1} = 13122 \quad \therefore 3^{n-1} = 6561$

but $6561 = 3^8 \quad \therefore 3^{n-1} = 3^8 \quad \therefore n-1 = 8 \quad \therefore n = 9$ Ans.

6561 | 3
2187 | 3
729 | 3
243 | 3
81

(b) Let the first term of the A.P. = a, the Common diff = d

$\therefore$ 1st term $= a$
2nd term $= a+d$
4th term $= a+3d$

$\therefore$ a, $(a+d)$, $(a+3d)$ form a G.P.

$\therefore \frac{a+3d}{a+d} = \frac{a+d}{a} \quad \therefore (a+d)^2 = a(a+3d)$

or $a^2+2ad+d^2 = a^2+3ad \quad \therefore d^2 = ad \quad \therefore d = a$

Ans. Q.E.D.

5. $f(x) = x^2-6x+5$

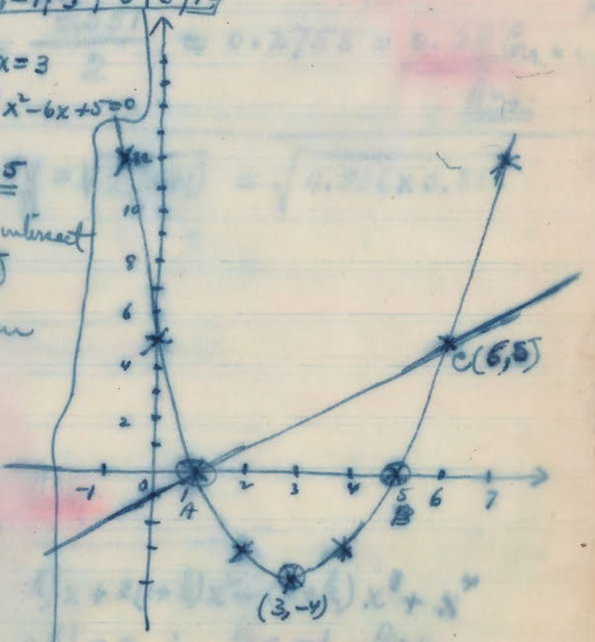
x	-1	0	1	2	3	4	5	6	7
y	12	5	0	-3	-4	-3	0	5	12

(a) the least value of the function is (-4) at $x=3$

(b) the roots of $x^2+5=6x$ are the same as the roots of $x^2-6x+5=0$
they are at A + B or $x=1$ and $x=5$

(c) Draw the graph of $y = x-1$ it will intersect the first curve at $A(1,0)$ & $C(6,5)$
the function $x^2-6x+5 < x-1$ between $x=1$ and $x=6$

(d) the points $A(1,0)$ & $C(6,5)$ lie on both curves $\therefore$ they satisfy both equations $y = x^2-6x+5$ and $y = x-1$
Hence $x=1$ and $x=6$ are the roots of $x^2-6x+5 = x+1$



Shamrock Secondary School

Additional Examination, September 1962

Subject : Mathematics
Class : 4th Year Secondary

Date : 14/9/1962
Time : 8:00 — 10:30

Attempt all questions:

1. (i) Solve the following simultaneous equations for x and y:

$$ax + by = c \quad \text{------} (1)$$

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{- - - -} (2)$$

(10 marks).

(ii) Solve the equation $4x^2 - 12x + 3 = 0$, giving the answer correct to two decimal places. (10 marks).

2. (i) Use logarithms to calculate by the shortest possible way, the value of $\sqrt{x^2+x}$, when $x = 4.836$. (10 marks).

(ii) When $(1-2x+x^2)$ is multiplied by $(1-kx+x^2)$ the coefficient of x^2 is zero. Find the value of k. (10 marks).

3. (i) Find the 50th term and the sum of the first 100 terms of the arithmetical progression whose first term is 20 and whose 3rd term is 21. (10 marks).

(ii) Find the first + sixth terms of a geometrical progression whose common ratio is $\frac{1}{2}$ when the first four terms add up to $18\frac{3}{4}$. (10 marks).

4. The cost of turfing a piece of lawn for a tennis court at 10 shillings per square yard is £12 less than 12 times the cost of fencing it all round at 5 shillings per yard. If the lawn had been 12 ft. longer it would have been twice as long as it is wide. Find the dimensions of the lawn in yards. (20 marks).

5. Draw the graph of $x^2 - 2x$ between $x = -2$ and $x = 4$.
From your graph solve approximately the equation $x^2 - 2x = 1.6$ With the help of a further graph, solve approximately the equation $x^2 - 2x = x + 1$

Solution to Conditional Exam, 14th September 1962

4th year Secondary.

$$\left.\begin{array}{l} ax+by=c \quad ----(1) \\ bx+ay=ab \quad ----(2) \end{array}\right\}$$

Multiply eqn. (1) by a, + eqn. (2) by b, + you get

$$\left.\begin{array}{l} a^2x + aby = ac \quad ---(3) \\ b^2x + aby = b^2 \quad ---(4) \end{array}\right\}$$ subtracting (4) from (3), we get:

$(a^2-b^2)x = ac - ab^2$ $\quad \therefore x = \dfrac{a(c-b^2)}{a^2-b^2}$

again multiply eqn. (1) by b and eqn. (2) by a, + you get:

$$\left.\begin{array}{l} abx+b^2y=bc \quad ---(5) \\ abx+a^2y=a^2b \quad ---(6) \end{array}\right\}$$ subtracting (5) from (6), we get:

$(a^2-b^2)y = a^2b - bc$ $\quad \therefore y = \dfrac{b(a^2-c)}{a^2-b^2}$

$$\left.\begin{array}{l} x = \dfrac{a(c-b^2)}{a^2-b^2} \\ y = \dfrac{b(a^2-c)}{a^2-b^2} \end{array}\right\}$$ **Ans.**

(ii) $4x^2-12x+3=0$ $\quad \therefore x = \dfrac{12 \pm \sqrt{144-4\times4\times3}}{2\times4} = \dfrac{12\pm\sqrt{96}}{8}$

$\therefore x = \dfrac{12\pm4\sqrt{6}}{8} = \dfrac{3\pm\sqrt{6}}{2}$ ~~$x_1 = \frac{3+\sqrt{6}}{2} = \frac{3+2}{2}$~~

$\therefore x_1 = \dfrac{3+\sqrt{6}}{2} = \dfrac{3+2.449}{2} = \dfrac{5.449}{2} = 2.7245 =$ **2.72** Correct to 2 dec. pl.

$x_2 = \dfrac{3-\sqrt{6}}{2} = \dfrac{3-2.449}{2} = \dfrac{0.551}{2} = 0.2755 =$ **0.28** Corr. " " **Ans.**

2. (i) $x = 4.836$, let $\sqrt{x^2+x} = y$ $\therefore y = \sqrt{x(x+1)} = \sqrt{4.836\times5.836}$

$\log 4.836 = 0.6844$

$\log 5.836 = 0.7661$

$2\log y = 1.4505$

$\log y = 0.72525$

$y = $ **5.311** or **5.31** **Ans.**

(ii) By actual multiplication:

$(1-2x+x^2)(1-kx+x^2) = 1-(2+k)x+2(1+k)x^2-(2+k)x^3+x^4$

$\therefore$ the coefficient of x^2 is $2(1+k)=0 \quad \therefore$ **$k=-1$** **Ans.**

3.(i) 1st term $a = 20$ ---- ①

2nd $\therefore a + d = 21$ ---- ②

subtract ① from ②, $\therefore d = 1$ $\therefore d = \frac{1}{2}$

Now $a = 20$, $d = \frac{1}{2}$, $l_{50} = ?$, $S_{100} = ?$

$l_{50} = a + (n-1)d = 20 + 49 \times \frac{1}{2} = 20 + 24\frac{1}{2} = 44\frac{1}{2}$ Ans. 1

$S_{100} = \frac{n}{2}\{2a + (n-1)d\} = \frac{100}{2}\{2\times 20 + (100-1)\times\frac{1}{2}\} = 50(40 + 49\frac{1}{2})$

$= 50 \times \frac{179}{2} = 4475$ Ans. 2

(ii) $r = \frac{1}{2}$, $S_4 = 18\frac{3}{4}$; $a = ?$, $l_6 = ?$ | $S_n = \frac{a(1-r^n)}{1-r}$

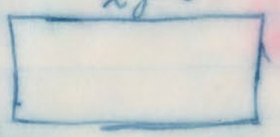
$\therefore S_4 = \frac{a(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$ $\therefore$ $18\frac{3}{4} = \frac{a(1-\frac{1}{16})}{\frac{1}{2}}$ or $\frac{75}{4} = 2a(\frac{15}{16})$

$\therefore a = \frac{75 \times 16}{4 \times 2 \times 15}$ or $a = 10$ Ans. 1

$l_6 = ar^5 = 10(\frac{1}{2})^5 = \frac{10}{32}$ Ans. 2

4. Let the length be $= x$ yds

" " width $= y$ yds.



$\therefore$ Cost of turfing $= 10xy$ shillings $= £\frac{xy}{2}$

also cost of fencing $= 5 \times 2(x+y)$ shillings $= £(x+y)$

$\therefore \frac{xy}{2} + 12 = 12 \times \frac{x+y}{2}$ ---- ①

Now $12\text{ft} = \frac{12}{3}\text{yds} = 4\text{ yds}$

$\therefore x + 4 = 2y$ ---- ②

from ①, $xy + 24 = 12x + 12y$ $\therefore (2y-4)y + 24 = 12(2y-4) + 12y$

from ②, $x = 2y - 4$ $\therefore 2y^2 - 4y + 24 = 24y - 48 + 12y$

or $2y^2 - 40y + 72 = 0$ or $y^2 - 20y + 36 = 0$ $\therefore (y-2)(y-18) = 0$

$\therefore y = 18$ or $y = 2$ (inadmissible)

$\therefore x = 2y - 4 = 2\times 18 - 4 = 32$ yds.

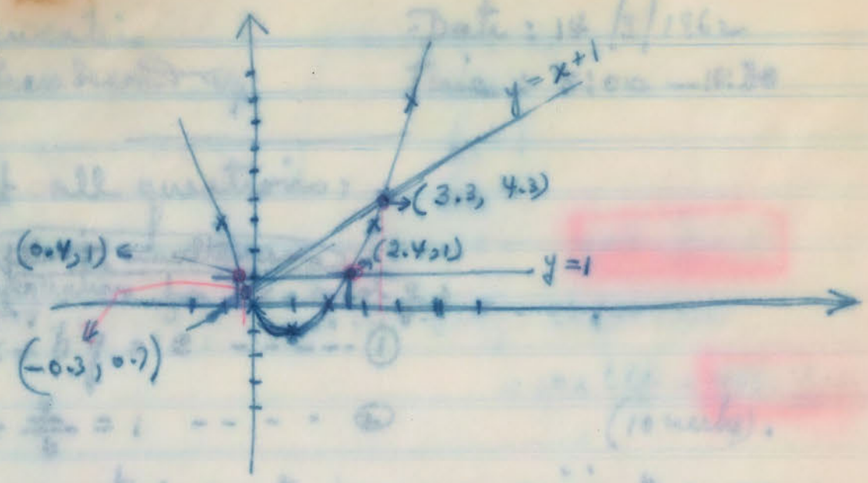
$\therefore$ length $x = 32$ yds, width $y = 18$ yds. Ans

Solution to Conditional Exam Qst. Sept. 1962.

4th Year Algebra

5. $f(x) = x^2 - 2x$

x	f(x)
-1	3
0	0
1	-1
2	0
3	3
4	8



∴ the solution of the equation $x^2 - 2x = 1$ gives $x_1 = -0.4$ and $x_2 = 2.4$ } Ans.

also solution of the equation $x^2 - 2x = x + 1$ gives:

$$x_1 = \frac{3+\sqrt{13}}{2} = 3.303$$

$$x_2 = \frac{3-\sqrt{13}}{2} = -0.303$$

} Ans.

Shamash Secondary School
Conditional Examination, Sept.1962.

Subject: Mathematics
Class: 4th Year Secondary

Date: 14/9/1962
Time: 8:00-10:30

Attempt all questions:

1. (i) Solve the following simultaneious equations for x and y :

$ax + by = c$ ..... (1)

$\frac{x}{a} + \frac{y}{b} = 1$ ..... (2) (10 marks).

(ii) Solve the equation $4x^2 - 12x + 3 = 0$, giving the answer correct to two decimal places. (10 marks).

2. (i) Use logarithms to calculate by the shortest possible way, the value of $\sqrt{x^2 + x}$, when $x = 4.836$. (10 marks)

(ii) When $(1-2x + x^2)$ is multiplied by $(1-kx+x^2)$ the coefficient of x^2 is zero. Find the value of k. (10 marks)

3. (i) Find the 50th term and the sum of the first 100 terms of the arithmetical progression whose first term is 20 and whose 3rd term is 21. (10 marks).

(ii) Find the first and sixth terms of a geometrical progression whose common ratio is $\frac{1}{2}$ when the first four terms add up to $18\frac{3}{4}$. (10 marks).

4. The cost of turfing a piece of lawn for a tennis court at 10 shillings per square yard is £ 12 less than 12 times the cost of fencing it all round at 5 shillings per yard. If the lawn had been 12 ft. longer it would have been twice as long as it is wide. Find the dimensions of the lawn in yards. (20 marks).

4. Draw the graph of x^2-2x between $x = -2$ and $x=4$.

From your graph solve approximately the equation $x^2-2x = 1$. With the help of a further graph, solve approximately the equation $x^2-2x = x + 1$. (20 marks).

(iii) From the curve of x^3 and $7x+6$, we have

$x^3 > 7x+6$

when $-3 < x < -1$ Ans.

also when $x > 3$ Ans.

13

Solutions to Algebra Final Exam, 4th year, 20 June 1942

1 (i) $\frac{x+5}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$ ... $\therefore x+5 = A(x+1) + B(x-1)$ [illegible]

Let $x = 1$, then $6 = 2A$ $\therefore A = 3$ } **Ans.**
Let $x = -1$, then $4 = -2B$ $\therefore B = -2$ }

8

(ii) Method 1: By the remainder & factor theorem, when $x = 2$, then $x-2 = 0$ and since $x-2$ is a factor of the expression $x^3 + px^2 + qx + 42$,
then $2^3 + p(2)^2 + q(2) + 42 = 0$ $\therefore 4p + 2q = -50$ or $2p + q = -25$
also $3^3 + p(3)^2 + q(3) + 42 = 0$ $\therefore 9p + 3q = -69$ or $3p + q = -23$
Solving the two equations simultaneously, we get

8

$p = 2$ and $q = -29$ Ans. Then the expression is $x^3 + 2x^2 - 29x + 42$
But $(x-2)(x-3) = x^2 - 5x + 6$ & the third factor is gotten by division as follows:

$$\begin{array}{l|l} x^3 + 2x^2 - 29x + 42 & x^2 - 5x + 6 \\ x^3 - 5x^2 + 6x & x + 7 \\ \hline 7x^2 - 35x + 42 & \\ 7x^2 - 35x + 42 & \\ \hline \end{array}$$

$\therefore$ the third factor is $(x+7)$ Ans. II

Method 2: By Division

$$\begin{array}{l|l} x^3 + px^2 + qx + 42 & x-2 \\ x^3 - 2x^2 & x^2 + (p+2)x + (2p+q+4) \\ \hline (p+2)x^2 + qx + 42 & \\ (p+2)x^2 - 2(p+2)x & \\ \hline [q + 2(p+2)]x + 42 \text{ or} & \\ (2p+q+4)x + 42 & \\ (2p+q+4)x - 2(2p+q+4) & \\ \hline \end{array}$$

Remainder $= 4p + 2q + 50 = 0$ or $2p + q = -25$

Also by actually dividing the same expression by $x-3$, we get $3p + q = -23$ which leads to the same solution. Q.E.D.

---

2 (i) $3\sqrt{x-1} + \sqrt{2x-3} = 2\sqrt{x+2}$ or $3\sqrt{x-1} = 2\sqrt{x+2} - \sqrt{2x-3}$ squaring,

8

$9(x-1) = 4(x+2) + (2x-3) - 4\sqrt{(x+2)(2x-3)}$ or
$4\sqrt{2x^2+x-6} = 14 - 3x$ squaring again, $16(2x^2+x-6) = 196 - 84x + 9x^2$
or $23x^2 + 100x - 292 = 0$ $\therefore (23x + 146)(x-2) = 0$ $\therefore x = 2$ Ans. 1
or $x = -\frac{146}{23}$ = Ans. 2 But Ans. 2 should be rejected as an extraneous root since it does not verify the original equation.

8

(ii) $\sqrt[3]{x-5} = \sqrt{x-6}$ or $(x-5)^{\frac{1}{3}} = (x-6)^{\frac{1}{2}}$ Raising both sides to the 6^{th} power, we get $(x-5)^2 = (x-6)^3$ or $(x-5)^3 - (x-5)^2 = 0$

or $(x-5)^2[(x-5)-1] = 0$ or $(x-5)^2(x-6) = 0$ $\therefore x = 5$ Ans. I, $+ x = 6$ Ans. II } both roots satisfy the original equation

8

(iii) $3x^{-1} - 10x^{-\frac{1}{2}} + 3 = 0$ let $x^{-\frac{1}{2}} = y$ $\therefore x^{-1} = y^2$

$\therefore 3y^2 - 10y + 3 = 0$ or $(3y-1)(y-3) = 0$ $\therefore y = 3$ or $y = \frac{1}{3}$

when $y = 3$, then $x^{-\frac{1}{2}} = 3$ or $x^{-1} = 9$ or $\frac{1}{x} = 9$ or $x = \frac{1}{9}$ Ans. I

when $y = \frac{1}{3}$, then $x^{-\frac{1}{2}} = \frac{1}{3}$ or $x^{-1} = \frac{1}{9}$ or $\frac{1}{x} = \frac{1}{9}$ or $x = 9$ Ans. II

both roots satisfy the original equation

3. (i) (a) $N^{\frac{1}{x^2-3x+9}} = \sqrt[7]{N}$ or $N^{\frac{1}{x^2-3x+9}} = N^{\frac{1}{7}}$

3 $\therefore \frac{1}{x^2-3x+9} = \frac{1}{7}$ $\therefore x^2 - 3x + 9 = 7$ or $x^2 - 3x + 2 = 0$ or

$(x-2)(x-1) = 0$ or $x = 2$ or $x = 1$ } Ans. both roots satisfy the original equation

3 (b) $\sqrt{5^2 - 4^2} = \sqrt[x]{81}$ or $3 = 3^{\frac{4}{x}}$ $\therefore \frac{4}{x} = 1$ $\therefore x = 4$ Ans.

(ii) $y^{2x} - 5y^x + 6 = 0$ $\therefore (y^x - 2)(y^x - 3) = 0$

2 $\therefore y^x = 2$ $\therefore x \log y = \log 2$ $\therefore x = \frac{\log 2}{\log y} = \frac{\log 2}{\log 10} = \frac{1}{2}\log 2$ Ans. 1

or $y^x = 3$ $\therefore x \log y = \log 3$ $\therefore x = \frac{\log 3}{\log y} = \frac{\log 3}{\log 10} = \frac{1}{2}\log 3$ Ans. 2

(iii) $x = \sqrt[7]{\frac{(6.0104)^2 \times (0.00003012)^3}{4020 \times (3019)^2}}$

6

$\log 0.0104 = \bar{2}.0170$	$2\log 0.0104 = \bar{4}.0340$	$\log 4020 = 3.6042$
$\log 0.00003012 = \bar{5}.4789$	$3\log 0.00003012 = \bar{14}.4367$	$2\log 3019 = 6.9598$
$\log 4020 = 3.6042$	$\log$ Numerator $= \bar{18}.4707$	$\log$ Denomin $= 10.5640$
$\log 3019 = 3.4799$	$\log$ Denominator $= 10.5640$	
	$7\log x = \bar{29}.9067$	
	$\log x = \bar{5}.98667 = \bar{5}.9867$ Correct to 4 dp.	
	$\therefore x = 9.699 \times 10^{-5}$ or 0.00009699	

Ans.

Let the x mi/hr. = rate of rowing in still water
+ Let the y mi/hr. = rate of Current.

13

$\therefore \frac{2}{\frac{x}{2}+y} + \frac{2}{\frac{x}{2}-y} = 1\frac{2}{3}$ or $\frac{4}{x+2y} + \frac{4}{x-2y} = \frac{5}{3}$ or $12(x+2y)+12(x-2y) = 5(x^2-4y^2)$

or $24x = 5x^2 - 20y^2$ ----- ①

also $\frac{2}{x+y} + \frac{2}{x-y} = \frac{2}{3}$ or $3(x-y)+3(x+y) = (x^2-y^2)$

or $6x = x^2 - y^2$ ----- ②

Multiply equation ② by 20 + subtract, hence

$120x = 20x^2 - 20y^2$ ----- ③

$24x = 5x^2 - 20y^2$ ----- ④

$96x = 15x^2$ $\therefore 32 = 5x$ $\therefore x = 0$ + $x = \frac{32}{5}$ Ans. I

Now $y^2 = x^2 - 6x$ or $y = \sqrt{x^2-6x}$ or $y = \sqrt{\frac{1024}{25} - \frac{192}{5}}$ or $y = \sqrt{\frac{1024-960}{25}}$

or $y = \frac{1}{5}\sqrt{64}$ or $y = \frac{8}{5}$ Ans. II

∴ the rate of rowing in still water $= \frac{32}{5} = 6.4$ miles/hr.

and the ∴ " Current $= \frac{8}{5} = 1.6$ miles/hr. } Ans.

---

5. (i) To get the first stone in the box the boy has to travel $1+1 = 2$ yds. To get the second stone in the box, the boy has to travel $2+2 = 4$ yds. + so on.

∴ total distance travelled to get 10 stones in the box =

$2+4+6+\ldots$ $\therefore S_{10} = \frac{n}{2}\{2a+(n-1)d\} = \frac{10}{2}\{4+9\times 2\}$

$= 5 \times 22 = 110$ yards.

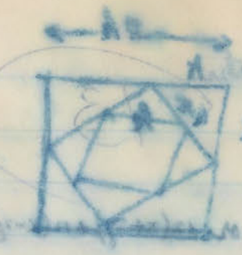
8

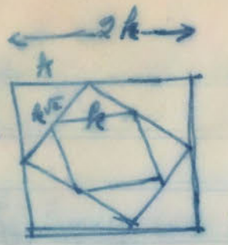
To get the first n stones: Ans. I

$S = \frac{n}{2}\{2\times 2 + (n-1)\times 2\} = \frac{n}{2}(2+2n) = n(n+1)$ Ans. II

To get the first $(n-1)$ stones:

$S = \frac{n-1}{2}\{2\times 2 + (n-2)\times 2\} = \frac{n-1}{2}(2n) = n(n-1)$ Ans. III





8)

(ii) The side of the first square = $2k$

" " " second square = $k\sqrt{2}$

" " " third square = $\frac{k\sqrt{2}}{2}\sqrt{2} = k$

and so on ∴ the sum of the sides of all the squares

equals $= 4(2k + k\sqrt{2} + k + \cdots \text{ to infinity})$

$$\therefore \underset{n\to\infty}{S_n} = 4\left[\frac{2k}{1-\frac{1}{\sqrt{2}}}\right]$$ as the brackets a infinite geometrical progression whose in which:

$a = 2k$, $r = \frac{k\sqrt{2}}{2k} = \frac{1}{\sqrt{2}}$ and $n = \infty$

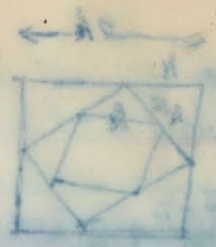
where $S = \frac{a}{1-r}$

$$\therefore \underset{n\to\infty}{S_n} = 4\left(\frac{2k\sqrt{2}}{\sqrt{2}-1}\right)$$

$$\therefore \underset{n\to\infty}{S_4} = \frac{8k\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = 8k\sqrt{2}(\sqrt{2}+1)$$ Ans. A.13.

$= 8 \times 18\sqrt{2}(\sqrt{2}+1) = 144\sqrt{2}(\sqrt{2}+1) = 288 + 144\sqrt{2}$ inches Ans.

6. (i)



September 14th, 1961

$4a^2(a-1)-9a+9 = 4a^2(a-1)-9(a-1) = (a-1)(4a^2-9)$

$= (a-1)(2a-3)(2a+3)$ Ans.

(vii) $\frac{7A-3B}{5A+6B} = ?$ given $\frac{A}{B}=11$, $\frac{7A-3B}{5A+6B} = \frac{7\frac{A}{B}-3}{5\frac{A}{B}+6} = \frac{7\times11-3}{5\times11+6} = \frac{74}{61}$ Ans.

(viii) $x - \left(\frac{2x}{5}+\frac{x}{6}\right) = \frac{x-\left(\frac{2x}{5}+\frac{x}{6}\right)}{x}$ is a fraction of the total

$= \frac{30x-12x-5x}{30x} = \frac{13x}{30x} = \frac{13}{30}$ Ans.

2. (i) $Ax^3+Bx^2+Cx+2=0$ when $x=1$, we get: $A+B+C+2=0$ ... ①

When $x=-1$ we get: $-A+B-C+2=0$ ... ②

When $x=\frac{3}{5}$ we get: $\frac{27}{125}A+\frac{9}{25}B+\frac{3}{5}C+2=0$ ... ③

from ① + ② we get: $2B+4=0$ or $B=-2$ Ans.I

from ③ $27A+125B+75C+250=0$ or $27A+125B+75C+250=0$ ... ④

substitute $B=-2$ in eqn. ② $\therefore A+C=0$ $\therefore A=-C$ → substitute in eqn. ④

we get $-27C-250+75C+250=0$ $\therefore 48C=0$ $\therefore C=0$ Ans.II

$\therefore A=C=0$ Ans.III. Hence $A=0$, $B=-2$, $C=0$ Ans.

(ii) $\sqrt{\frac{Ay-1}{Ax+1}} = \frac{y}{x}$, $A=?$ $\therefore \frac{Ay-1}{Ax+1} = \frac{y^2}{x^2}$ $\therefore Ax^2y-y^2 = Axy^2-x^2$

$\therefore Axy(x-y) = x^2+y^2$ $\therefore A = \frac{x^2+y^2}{xy(x-y)}$ Ans.

3. (i) $(729)^{\frac{5}{6}} = (3^6)^{\frac{5}{6}} = 3^5 = 243$ Ans.I

(ii) $\left(\frac{32}{3125}\right)^{-\frac{3}{5}} = \left(\frac{3125}{32}\right)^{\frac{3}{5}} = \left(\frac{5^5}{2^5}\right)^{\frac{3}{5}} = \left(\frac{5}{2}\right)^3 = \frac{125}{8} = 15\frac{5}{8} = 15.625$ Ans.II

$\frac{(343)^{\frac{5}{6}}}{(343)^{\frac{1}{2}}} = \frac{(7^3)^{\frac{5}{6}}}{(7^3)^{\frac{1}{2}}} = 7^{\frac{5}{2}-\frac{1}{2}} = 7^2 = 49$ Ans.I

(ii) $\sqrt[11]{0.0001} = 10^{-\frac{4}{11}}$ Ans.II

(iii) Let $x = \sqrt[7]{\frac{(0.00561)^2}{1.008}} =$

$\log 0.00561 = \bar{3}.7490$

$\log 1.008 = 0.0033$

$2\log 0.00561 = \bar{5}.4980$

$\log 1.008 = 0.0033$

$\frac{1}{7}\log x = \bar{5}.4947$

$\log x = \bar{1}.35638\ldots = \bar{1}.3564$ Correct to 4 dec. pl.

$\therefore x = 0.2272$ Ans.

September 13 61

$S = \frac{n}{2}(a+l)$ ∴ $35 = \frac{7}{2}(a+l)$. also: $l - a = 18$ or $l = a+18$

∴ $35 = \frac{7}{2}(a+a+18)$ or $10 = 2a+18$ or $2a = -8$

∴ $a = -4$ ∴ $l = a+18$ or $l = 14$.

but, $l = a+(n-1)d$ or $d = \frac{l-a}{n-1}$ $\frac{14+4}{6} = 3$. Hence the seven numbers are:

$-4, -1, 2, 5, 8, 11, 14$ Ans.

(ii) $ar^3 + 6ar^4 = ar^2$ Dividing by ar^2, we get: $r + 6r^2 = 1$ or

$6r^2 + r - 1 = 0$ or $(3r-1)(2r+1) = 0$ or $r = \frac{1}{3}$ and $r = -\frac{1}{2}$

Ans. i

$ar = 16$ and $r = -\frac{1}{2}$ ∴ $-\frac{a}{2} = 16$ ∴ $a = -32$

$S = \frac{a(1-r^n)}{1-r} = \frac{-32\left[1-\left(-\frac{1}{2}\right)^6\right]}{1+\frac{1}{2}} = \frac{-32\left[1-\frac{1}{64}\right]}{\frac{3}{2}} = -\frac{32\times 2}{3}\left(\frac{63}{64}\right)$

$= -21$ Ans. ii

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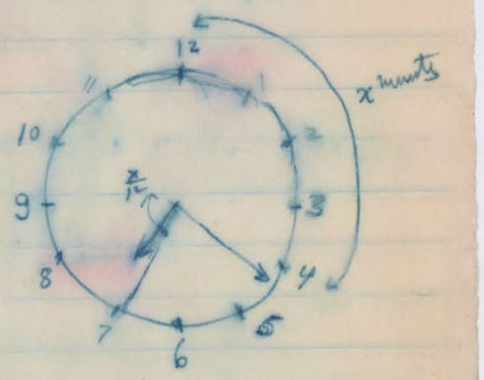
5. Let the time be x minutes after 7:

The Hour hand would have moved

$\frac{x}{12}$ divisions beyond No. 7



∴ $x + 15 = 35 + \frac{x}{12}$

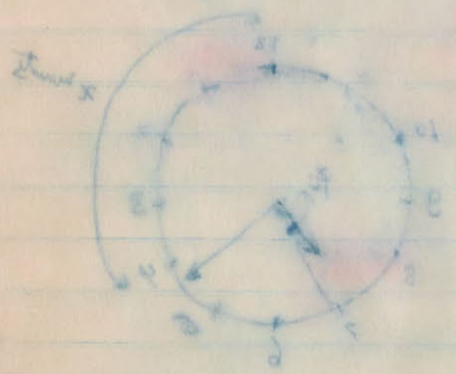
∴ $x - \frac{x}{12} = 20$

∴ $12x - x = 240$

∴ $11x = 240$

∴ $x = \frac{240}{11} = 21\frac{9}{11}$ minutes

∴ The time is $21\frac{9}{11}$ minutes past 7. Ans.



Shamash Secondary School
Conditional Examination, September 1961

Subject: Algebra
Class: 4th Year Secondary

Date 1/9/1961
Time: 8:00-10:30

Attempt all questions:

1. (i) Factor completely: $4a^2(a-1) - 9a+9$ (6 marks)

(ii) Find the numerical value of $\frac{7A-3B}{5A+6B}$, having given $\frac{A}{B} = 11$ (7 marks)

(iii) A basket contains oranges, lemons and apples. The total number of these fruits is X. There are $\frac{2X}{5}$ oranges and $\frac{X}{6}$ lemons. What fraction of the total is the number of apples? (7 marks)

2. (i) If X =1, X=-1 and $X=\frac{2}{3}$ satisfy the equation $Ax^3+Bx^2+Cx+2=0$, find the values of A,B and C. (10 marks)

(ii) Given that $\sqrt{\frac{Ay-1}{Ax+1}} = \frac{y}{x}$, obtain in its simplest form an expression for A in terms of x and y. (10 marks)

3. (i) Calculate the values of $(729)^{\frac{5}{6}}$, $(\frac{32}{3125})^{-\frac{3}{5}}$ and $(343)^{\frac{5}{6}} \div (343)^{\frac{1}{2}}$ (7 marks)

(ii) Express $\sqrt[11]{0.0001}$ as a power of 10. (6 marks)

(iii) Use logarithms to calculate the value of $\sqrt[7]{\frac{(0.00561)^2}{1.008}}$ correct to four decimal places. (7 marks)

4. (i) The sum of seven numbers which are in arithmetic progression is 35 and the difference between the first and the seventh is 18. Find the seven numbers. (8 marks)

(ii) The sum of the fourth term and six times the fifth term of a geometric progression is equal to the third term. Find the two possible values of the common ratio.
If the second term is 16 and the common ratio is negative, find the sum of the first six terms. (12 marks)

5. Find the time between 7 and 8 o'clock when the hands of a watch are separated by 15 minutes for the first time. (20 marks)

P.T.O.

Final Exam. in Algebra

June 12th 1961

Let x shillings = price of one gross. $\therefore \frac{12x}{144}$ pence $\left(= \frac{x}{12} \text{ pence}\right)$ = price of one pencil in pence

$\therefore \frac{20 \times 12}{\frac{x}{12}} = \frac{2880}{x}$ pence price of one score in pennies in 1st case.

$\therefore \frac{20 \times 12}{\frac{x}{12}} = \frac{2880}{x}$ = No. of pencils which can be bought for £1 in 1st case

$\therefore \frac{2880}{x} + 120 = \frac{2880 + 120x}{x}$ " " " " " " 2nd case

$\therefore \frac{240}{\frac{2880+120x}{x}} = \frac{240x}{120(24+x)} = \frac{2x}{24+x}$ pence = price of one pencil in pennies in 2nd case

$\therefore 20\left(\frac{2x}{x+24}\right) = \frac{40x}{x+24}$ pence = price of one score of pencils in 2nd case.

$\therefore \frac{5}{3}x - \frac{40x}{x+24} = 2 \quad \therefore 5x^2 + 120x - 120x = 6x + 144$ or

$5x^2 - 6x - 144 = 0 \quad \therefore (5x+24)(x-6) = 0 \quad \therefore x = -\frac{24}{5}$ to be discarded

or $x = 6$ Ans. Hence Price of one gross = 6 shillings Ans.

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2. (i) $\sqrt{a-x} + \sqrt{b+x} = \sqrt{a} + \sqrt{b}$ squaring, we have:

$a - x + 2\sqrt{(a-x)(b+x)} + b + x = a + 2\sqrt{ab} + b$ or $\therefore (a-x)(b+x) = a.b$

$\therefore a.b + (a-b)x - x^2 = a.b.$ or $x^2 - (a-b)x = 0$ or $x[x-(a-b)] = 0$

$\therefore x = 0$ Ans. I or $x = a - b$ Ans. II

(ii) $\left\{\frac{a^{p-q}}{\sqrt[p]{a^{p^2-pq}}} \times a^{-(p-q)}\right\}^n = \left\{\frac{a^{p-q}}{a^{q-p}} \times a^{2p-2q}\right\}^n = \left\{a^{p-q-q+p+2p-2q}\right\}^n =$

$= \left\{a^{4(p-q)}\right\}^n = a^{4n(p-q)}$ Ans.

---

3. (i) $\sqrt[7]{\frac{(0.0010\,21)^2 \times (4.003)^3}{(16.02)^5 \times (3.001)^4}}$

$\log 0.0010\,21 = \bar{3}.0090$

$\log 4.003 = 0.6024$

$\log 16.02 = 1.2046$

$\log 3.001 = 0.4772$

$2 \log 0.0010\,21 = \bar{6}.0180$

$3 \log 4.003 = 1.8072$

$\log \text{Num.} = \bar{5}.8252$

$\log \text{Den.} = 7.9318$

$7 \log x = \bar{13}.8934$

$\log x = \bar{2}.27048\ldots = \bar{2}.2705$ c.f. 4 d.p.

$x = 0.01864$ Ans.

$5 \log 16.02 = 6.0230$

$4 \log 3.001 = 1.9088$

$\log \text{Den.} = 7.9318$

Ans. ii - 1.5°024

P. T. O.

Solutions to Algebra

4^{th} Year 12/6/1961

(iii) Suppose it is required to find $\log_y N$ in terms of logarithms to the base x

$\therefore$ let $\log_y N = z \quad \therefore N = y^z \quad \therefore \log_x N = z \log_x y \quad \therefore z = \dfrac{\log_x N}{\log_x y}$

$\therefore \log_y N = \dfrac{\log_x N}{\log_x y} \quad \therefore \log_{17} 70.56 = \dfrac{\log_{10} 70.56}{\log_{10} 17}$

$\therefore \log_{17} 70.56 = \dfrac{1.8486}{1.2304} = \dfrac{18486}{12304} = 1.50243 \ldots = 1.5024$ Correct to 4 decimal places Ans.

---

4. (i) $14\frac{17}{20}$ = the sum of an Arithmetic progression whose No. of terms is 18 + whose common difference is $(-\frac{1}{20})$. Applying the formula $S = \frac{n}{2}\{2a + (n-1)d\}$

we get: $14\frac{17}{20} = \frac{18}{2}\{2a + 17(-\frac{1}{20})\}$ or $14\frac{17}{20} = 9(2a - \frac{17}{20})$

$\therefore 18a = 14\frac{17}{20} + \frac{9 \times 17}{20}$ or $18a = 14\frac{17}{20} + 7\frac{13}{20}$ or $18a = 22\frac{1}{2}$

or $18a = \frac{45}{2} \quad \therefore a = \frac{45}{36}$ or $a = \frac{5}{4} \quad \therefore l = a + (n-1)d$

$\therefore l_{18} = \frac{5}{4} + 17(-\frac{1}{20})$ or $l_{18} = \frac{25}{20} - \frac{17}{20}$ or $l_{18} = \frac{8}{20} = \frac{2}{5}$

$\therefore$ distance driven in the 1st minute $= a = \frac{5}{4}$ miles
and " " " last " $= l = \frac{2}{5}$ miles } Ans.

(ii) $S = 4(5^n - 1)$. Suppose $n = 1$, $\therefore S_1$ = the 1st term $= 4(5^1 - 1) = 16$

Again suppose $n = 2 \quad \therefore S_2 = 4(5^2 - 1) = 4 \times 24 = 96$ = 1st + 2nd terms

$\therefore$ 2nd term $= 96 - 16 = 80 \quad \therefore r = \frac{80}{16} = 5 \quad \therefore l_5 = ar^4 = 16(5)^4 = 16 \times 625$

$\therefore l_5 = 10000$ Ans. 1

Now if $S = 312496$, then $312496 = 4(5^n - 1)$ or $5^n = \frac{312496}{4} + 1$ or

$5^n = 78124 + 1$ or $5^n = 78125$ or $5^n = 5^7 \quad \therefore n = 7$ Ans. 2.

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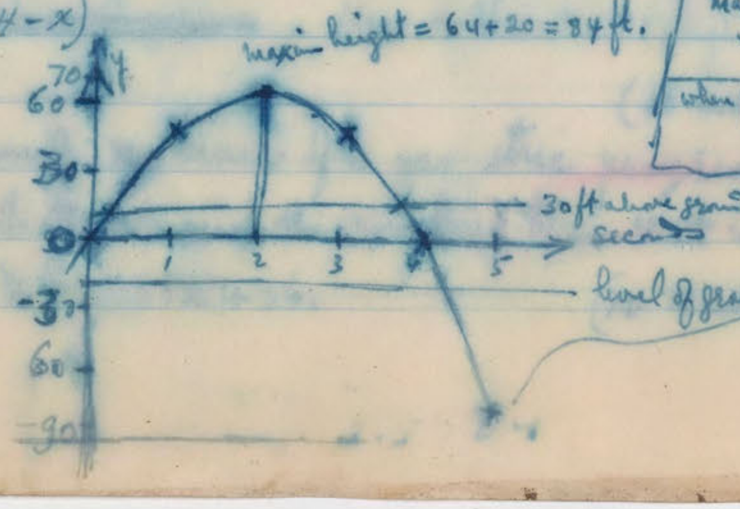
5. $y = 16x(4 - x)$

x	y
0	0
1	48
2	64
3	48
4	0
5	-80



maxm height above point A = 64 ft
" " " ground level = 64+20 = 84 ft

when $y = 48$, ... $x^2 - 4x + 3 = 0$
or $x = 1$ or $3 \quad \therefore 3 - 1 = 2$ seconds

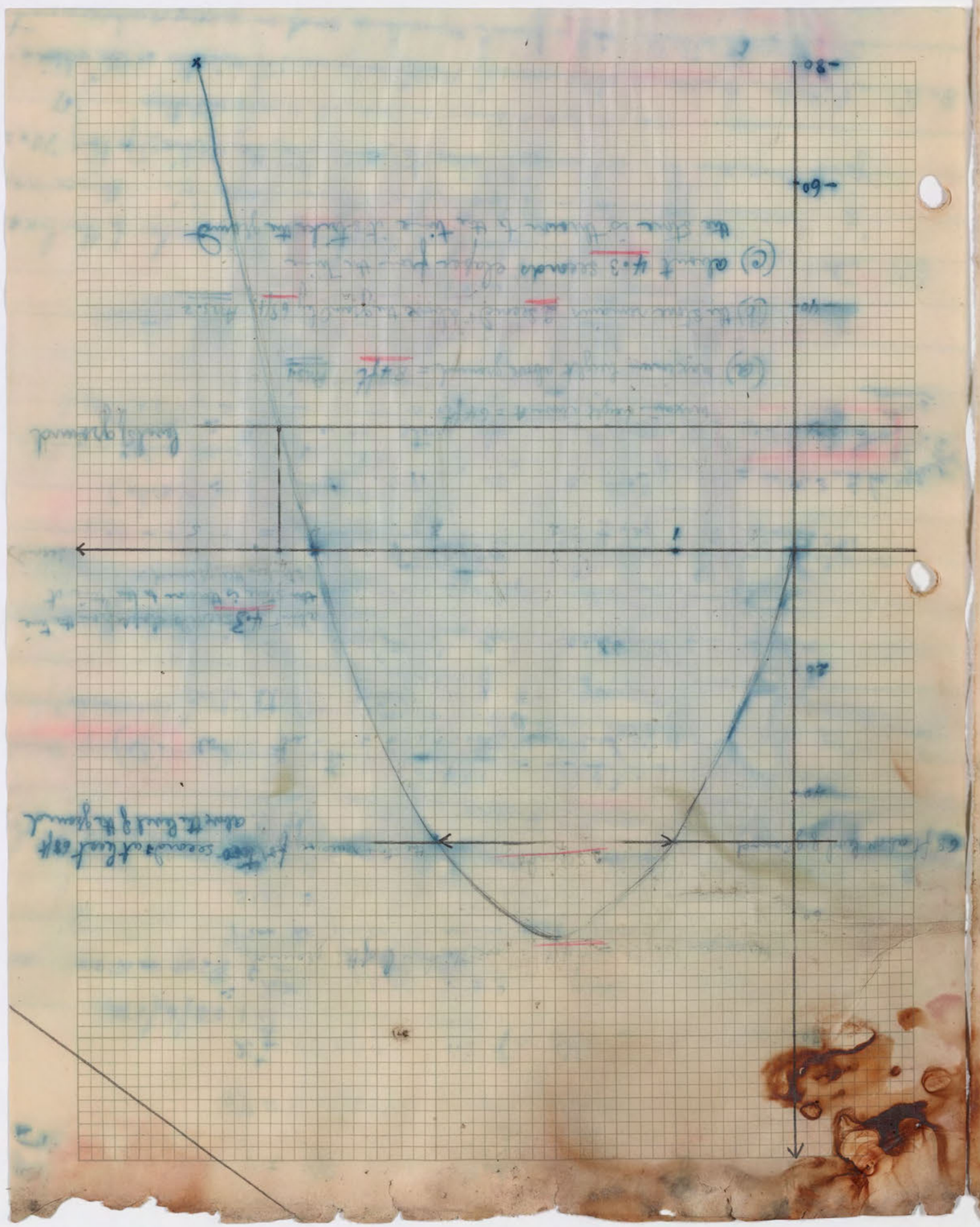
When the stone strikes the ground, $y = -20$

then $4x^2 - 16x + 5 = 0$

$\therefore x = \frac{16 \pm 4\sqrt{11}}{8} = 2 \pm \frac{1}{2}\sqrt{11}$

$= 4.5 \ldots$ seconds

or -0.5 to be rejected.



$x^2 - 4x + 3 = 0$ or $(x-1)(x-3) = 0$

∴ $x = 1$ or $x = 3$

From the graph we [illegible] that [illegible] the [illegible] second [illegible] 48 ft [illegible] above the ground.

∴ the [illegible] = [illegible] seconds. Ans.

(c) When the stone strikes the ground, $y = -20$ [illegible] substitute -20 for y and you get

$-20 = 16x(4-x)$ or $-20 = 64x - 16x^2$

$16x^2 - 64x - 20 = 0$ or $4x^2 - 16x - 5 = 0$

∴ $x = \frac{16 \pm \sqrt{256+80}}{8} = 2 \pm \frac{1}{2}\sqrt{21} = 2 \pm \frac{1}{2} \times 4.583$

∴ $x = 2 \pm 2.291$ or $x = 4.3$ or -0.3 to be neglected

∴ the time the stone will be in the air = 4.3 seconds. Ans.

Date: 12/6/1961

Time: 8:00 – 11:00

Attempt all questions.

1. How much a [illegible] pencil [illegible] a gross [illegible] more for a sovereign lowers the price [illegible] a score [illegible] (10 marks)

2. (i) Solve the equation $\sqrt{a-x} + \sqrt{b+x} = \sqrt{a} + \sqrt{b}$ (10 marks)

(ii) Simplify & express with positive indices: [illegible] (10 marks)

3. (i) Compute by logarithms the following:

$$\sqrt[7]{\frac{(0.0010.21)^2 \times (4.003)^3}{(16.02)^5 \times (3.001)^4}}$$ (10 marks)

(ii) Derive a formula for finding the logarithm of a number to the base 'y' having given tables of logarithms to the base 'x'. Hence use your common logarithmic tables to compute the value of $\log_{17} 70.56$. (10 marks)

4. (i) A man drives a distance of $14\frac{17}{20}$ miles in 18 minutes. He drives the longest stretch in the first minute; and in every subsequent minute the distance he drives is $\frac{1}{20}$ miles shorter than the distance driven in the previous minute. What are the distances driven in the first & last minutes. ~~Consider these distances as an arithmetical progression & solve~~. Use Arithmetic Progressions to solve your question. (10 marks)

(ii) The sum of n terms of a geometric progression is $4(5^n - 1)$. Find the fifth term and the number of terms that must be taken for their sum to be 312496. (10 marks)

Final Exam in Algebra Cont.

12/6/1961 June 1961

In the equation $y = 16x(4-x)$, 'y' represent the height in feet risen by a stone thrown vertically upward from a point A on the roof of a building 20 ft above the level of the ground; and 'x' represents the time in seconds taken by the stone to reach the height 'y' from that point.

Draw a graph between $x = 0$ and $x = 5$ showing the relationship between y and x. (Take 1 in. = 1 second and 20 ft respectively.)

From the graph, find

(a) the maximum height above the level of the ground reached by the stone,

(b) how long the stone remains at least 68 ft above the ground,

(c) how many seconds elapse from the time the stone was thrown to the time the stone strikes the ground.

(20 marks)

Shortish Secondary School

Final Examinations, June 1961.

Subject : Algebra
Class : 4th Year Secondary

Date : 12/6/1961
Time : 8:30-11:00 a.m.

Attempt all questions :

1. How much are pencils a gross when 120 more for a Sovereign lowers the price 2d. a score ? (20 marks).

2. (i) Solve the equation $\sqrt{a - x} + \sqrt{b + x} = \sqrt{a} + \sqrt{b}$. (10 marks).

(ii) Simplify and express with positive indices :

$$\left\{ \frac{a^{p-q}}{\sqrt[q]{a^{q^2-pq}}} \times a^{z(p-q)} \right\}^n$$ (10 marks).

3. (i) Compute by logarithms the following :

$$\sqrt[7]{\frac{(0.001021)^2 . (4.003)^3}{(16.02)^5 . (3.001)^4}}$$ (10 marks).

(ii) Derive a formula for finding the logarithms of a number to the base 'y' having given tables of logarithms to the base 'x'. Hence use your common logarithmic tables to compute the value of $\log_{17} 70.56$. (10 marks).

4. (i) A man drives a distance of $14\frac{17}{20}$ miles in 18 minutes. He drives the longest stretch in the first minute; and in every subsequent minute the distance he drives is $\frac{1}{20}$ miles shorter than the distance driven in the previous minute. What are the distances driven in the first and last minutes ? Use arithmetic progressions to solve the question. (10 marks).

(ii) The sum of n terms of a geometric progression is $4(5^n - 1)$. Find the fifth term and the number of terms that must be taken for their sum to be 312496. (10 marks).

5. In the equation $y = 16x(4 - x)$, y represents the height in feet risen by a stone thrown vertically upward from a point A on the roof of a building 20 ft. above the level of the ground; and x represents the time in seconds taken by the stone to reach the height y from that point.
Draw a graph between $x = 0$ and $x = 5$ showing the relationship between y and x. (Take 1 in. = 1 second and 20 ft. respectively).
From the graph find :
(a) the maximum height above the level of the ground reached by the stone,
(b) how long the stone remains at least 68 ft. above the ground,
(c) how many seconds elapse from the time the stone was thrown to the time the stone strikes the ground. (20 marks).

Shenash Secondary School

3rd Quarter Examination

Subject : Algebra
Class : 4th Secondary

Date : 17/3/1961.
Time : 8:30--10:00 a.m.

Attempt all Questions:

1. A boy can swim at v m.p.h. in still water. When he swims downstream the current increases his speed by u m.p.h., and when he swims upstream the current decreases his speed by u m.p.h. The difference in his time to swim one mile downstream and one mile upstream is t hours. Find a formula for t in terms of v and u, and find v if $u = 2,\ t = \frac{4}{5}$.

2. A dealer bought a horse, expecting to sell it again at a price that would have given him 10 per cent. profit on his purchase; but he had to sell it for £50 less than he expected, and he then found that he had lost 15 per cent. on what it cost him. What did he pay for the horse ?

3. At what time between 12 o'clock and 1 o'clock are the two hands of a watch at right angles for the second time?

4. By lowering the price of eggs and selling them one penny per egg cheaper, a man finds that he can sell 5 more than he used to do for 5s. At what price per egg did he sell them at first?

5. A cistern can be filled by two pipes running together in 24 minutes. The larger pipe would fill the cistern in 20 minutes less than the smaller one. Find the time taken by each.

Shenash Secondary School

3rd Quarter Examination

Subject : Algebra
Class : 4th Secondary

Date : 17/3/1961.
Time : 8:30--10:00 a.m.

Attempt all Questions:

1. A boy can swim at v m.p.h. in still water. When he swims downstream the current increases his speed by u m.p.h., and when he swims upstream the current decreases his speed by u m.p.h. The difference in his time to swim one mile downstream and one mile upstream is t hours. Find a formula for t in terms of v and u, and find v if $u = 2, t = \frac{4}{5}$.

2. A dealer bought a horse, expecting to sell it again at a price that would have given him 10 per cent. profit on his purchase; but he had to sell it for £50 less than he expected, and he then found that he had lost 15 per cent. on what it cost him. What did he pay for the horse ?

3. At what time between 12 o'clock and 1 o'clock are the two hands of a watch at right angles for the second time?

4. By lowering the price of eggs and selling them one penny per egg cheaper, a man finds that he can sell 5 more than he used to do for 5s. At what price per egg did he sell them at first?

5. A cistern can be filled by two pipes running together in 24 minutes. The larger pipe would fill the cistern in 20 minutes less than the smaller one. Find the time taken by each.

Shamash Secondary School
Final Examination, June 1960.

Subject: Algebra
Class: 4th Year Secondary

Date: 17/6/1960
Time: 8:00-11:00

All questions are to be attempted.

1. (a) If $X = \frac{2Y-1}{3Y-4}$, and $Y = \frac{Z+1}{Z-1}$, find Z in terms of X. (8 marks)

(b) Prove that, if a + b = C, and none of these quantities is zero, the expression

$$\frac{1}{a^2 + b^2 - c^2} + \frac{1}{b^2 + c^2 - a^2} + \frac{1}{c^2 + a^2 - b^2}$$

is equal to zero. (8 marks)

2. (i) Find the value of b for which the expression $X^3 - 2 - b(X-1)$ is equal to Zero when X = 2. (5 marks)

(ii) Factorize the expression for this value of b, and find the other values of X for which the expression is Zero.
(12 marks)

3. A train left station P at 10 a.m. on a non-stop run of 300 miles to station Q where it was due to arrive at 4:15 p.m. At a station B some miles from Q it was $3\frac{3}{4}$ minutes behind the Scheduled time. But by travelling from B to Q at 60 miles per hour the train arrived at its destination on time.
How far is it from B to Q ? (17 marks)

4. (a) Compute by logarithms the following expression:

$$\sqrt[7]{\frac{1.004^2 \times 0.000401^3}{516.2 \times 2.003^5}}$$ (8 marks)

(b) Use logarithms to solve the equation $4^{2x} - 8 \times 4^x + 12 = 0$
(8 marks)

5. (a) The 21st term of an arithmetical progression is $2\frac{1}{2}$ times the 8th term, and the arithmetic mean of the 5th and 13th terms is 29. Find the sum of the first 15 terms. (8 marks)

(b) Prove that in any Geometric series the sum of the 4th, 5th, and 6th terms is the Geometric mean of the sum of the 1st, 2nd, and 3rd terms and the sum of the 7th, 8th, and 9th terms. (8 marks)

6. (a) Draw the graph of $Y = X^2 - 3X + 2$ for values of X between -2 and 5.
(6 marks).

(b) Use your graph to solve the equations:
(i) $X^2 - 3X + 2 = 0$ (6 marks)
(ii) $X^2 - 3X - 4 = 0$ (6 marks)

Final examination, [illegible]

: Algebra

: 4th year Secondary

Date: 17/6/1960

Time: 8:00 - 11:00

All questions are to be attempted.

1. (a) If $x = \frac{2y-1}{3y-4}$, and $y = \frac{3+1}{3-1}$, find z in terms of x. (8 marks)

(b) Prove that, if $a+b=c$, and none of these quantities is zero, the expression

$$\frac{1}{a^2+b^2-c^2} + \frac{1}{b^2+c^2-a^2} + \frac{1}{c^2+a^2-b^2}$$

is equal to zero. (8 marks)

2. (i) Find the value of b for which the expression $x^3 - 2x^2 - b(x-1)$ is equal to zero when $x = 2$. (5 marks)

(ii) Factorize the expression for this value of b, and find the values of x for which the expression is zero. (12 marks)

3. A train left station P at 10 a.m. on a non-stop run of 300 miles to station Q where it was due to arrive at 4:15 p.m. At a station R some miles from Q it was $3\frac{3}{4}$ minutes behind scheduled time. But by travelling from R to Q at 60 miles per hour the train arrived at its destination on time. How far is it from R to Q? (17 marks)

4. (a) Compute by logarithms the following expression:

$$\sqrt[7]{\frac{1.004^5 \times 0.000401^3}{516.2 \times 2.003^5}}$$ (8 marks)

(b) Use logarithms to solve the equation $4^{2x} - 8 \times 4^x + 1 = 0$. (6 marks)

(b) Find the 8th term and the arithmetic mean of the 5th and 11th terms is 29. Find the sum of the first 15 terms. (8 marks)

(b) Prove that in any Geometric series the sum of the 4th and 5th and 6th terms is the Geometric mean of the 1st, 2nd, and 3rd terms and the 7th, 8th, and 9th terms. (8 marks)

6. (a) Draw the graph of $y = x^2 - 3x + 2$ for values of x between -2 and 5. (6 marks)

(b) Use your graph to solve the equations:

(i) $x^2 - 3x + 2 = 0$ (6 marks)

(ii) $x^2 - 3x - 4 = 0$ (6 marks)

All questions are to be attempted

1. (a) Resolve into factors: (i) $3(2x-5)^2 - y(2x-5) - 2y^2$

(ii) $5(x-y)z^2 + (x-y)^3$

(b) If $k = \frac{2ab}{a+b}$, find: (i) "a" in terms of "b" and "k" (2 marks)

(ii) "b" in terms of "a" and "k" (2 marks)

(iii) the value of $\sqrt{\frac{k-4a}{k-5b}}$ in terms of "a" and "b".

$a = \frac{5bk}{4(5b-k)}$

$b = \frac{4ak}{5(4a-k)}$

$\frac{4a}{5b}$

2. Using tables, compute by logarithms the value of:

$\sqrt[3]{\frac{1.002 \times 37^2 \times 2.005}{40.03^2}}$ (10 marks)

3. (a) Solve for x the equation $3^{2x} - 28 \times 3^x + 27 = 0$

(b) A man in a speed-boat sees the flash of a gun fired at sea directly towards him and hears the report 20 seconds later. The boat is travelling to meet the ship at 30 miles per hour. Find the distance between the two when the shot was fired, taking the velocity of sound to be 1100 ft. per second. (20 marks)

4. Draw the graph of $y = 6+3x-x^2$ for values of x from -2 to 5 taking 1 inch as unit on the x-axis and $\frac{1}{2}$ inch as unit on the y-axis. From your graph find:

(i) the maximum value of y

(ii) the values of x between which the function $6+3x-x^2$ is positive.

5. (a) The third term of a series is $\frac{1}{4}$, $\frac{2}{10}$, $\frac{1}{20}$, $\frac{2}{5}$, ... Find its nth term and the sum of the first ten terms. (10 marks)

(b) [illegible]

SHAMASH SECONDARY SCHOOL

Final Examination, 1958-1959.

Subject : Algebra — Date : 28/5/1959
Class : 4th Year Secondary — Time : 8:00-10:30 a.m.

All questions are to be attempted.

1. (a) Resolve into factors: (i) $(7x + 8)^2 - 2(7x + 8) - 15$. (3 marks)

(ii) $2(x - y)^2 - 3x + 3y - 5$. (4 marks)

(iii) $a(a - 4) - b(b - 4)$. (3 marks)

(b) If $15(2x^2 - y^2) = 7xy$, and if x and y are both positive, find the ratio of x to y. Use the shortest possible way. (10 marks)

2(i) Using tables, compute by logarithms the value of :

$$\sqrt[5]{\frac{(0.004678)^2 \times 1.002}{(30.04)^3}}$$ (10 marks).

(ii) Given : log70 = 1.8451, log110 = 2.0414, log34.62 = 1.5394,

compute, without using tables, the value of : $\sqrt[3]{41503}$, correct to four sugnificant figures. (10 marks).

3. A certain alloy contains 6 parts by weight of a metal A and 5 parts by weight of a metal B; another alloy contains 1 parts by weight of A and 13 parts by weight of B. If these alloys are melted and mixed together, how many pounds of the second alloy must be mixed with 11 pounds of the first alloy to make a mixture which contains 40 per cent. of A ? (20 marks).

4. (a) The expression $2 - \frac{2^{n+1}}{3^n}$ is a formula for the sum of 'n' terms of a certain geometric series, n being any positive integer. Find the first term of the series, the common ratio, and the formula for the n-th term. (10 marks).

(b) The first and second terms of a series are 'a' and 'b' respectively. Find the n th term (i) if the series is an arithmetic series; (ii) if it is a geometric one. (10 marks).

5. (i) Taking $\frac{1}{2}$ in. as one unit on the x-axis and on the y-axis, plot the curve $y = \frac{3}{4}x^2$ for values of x between $x = -4$ and $x = 4$. (7 marks).

(ii) On the same axes of coordinates draw the graph of the equation $3x + 2y = 12$. (6 marks).

(iii) From the above graphs find two roots for the simultaneous equations $y = \frac{3}{4}x^2$ and $3x + 2y = 12$. Verify your graphical answers by solving algebraically. (7 marks).

Sum of 1st & 2nd terms

∴ Common ratio $r = \frac{4}{9} \div \frac{2}{3} = \frac{4}{9} \times \frac{3}{2} = \frac{2}{3}$ Ans. 2

∴ the n^{th} term $t_n = ar^{n-1} = \frac{2}{3} \times \left(\frac{2}{3}\right)^{n-1} = \left(\frac{2}{3}\right)^{n}$ Ans. 3

(b) (i) In the Arithmetic Series, the common difference: $d = b - a$

∴ the n^{th} term $t_n = a + (n-1)(b-a)$ Ans. 1

(ii) In the Geometric Series, the Common ratio: $r = \frac{b}{a}$.

∴ the n^{th} term: $t_n = a\left(\frac{b}{a}\right)^{n-1} = \frac{b^{n-1}}{a^{n-2}}$ Ans. 2

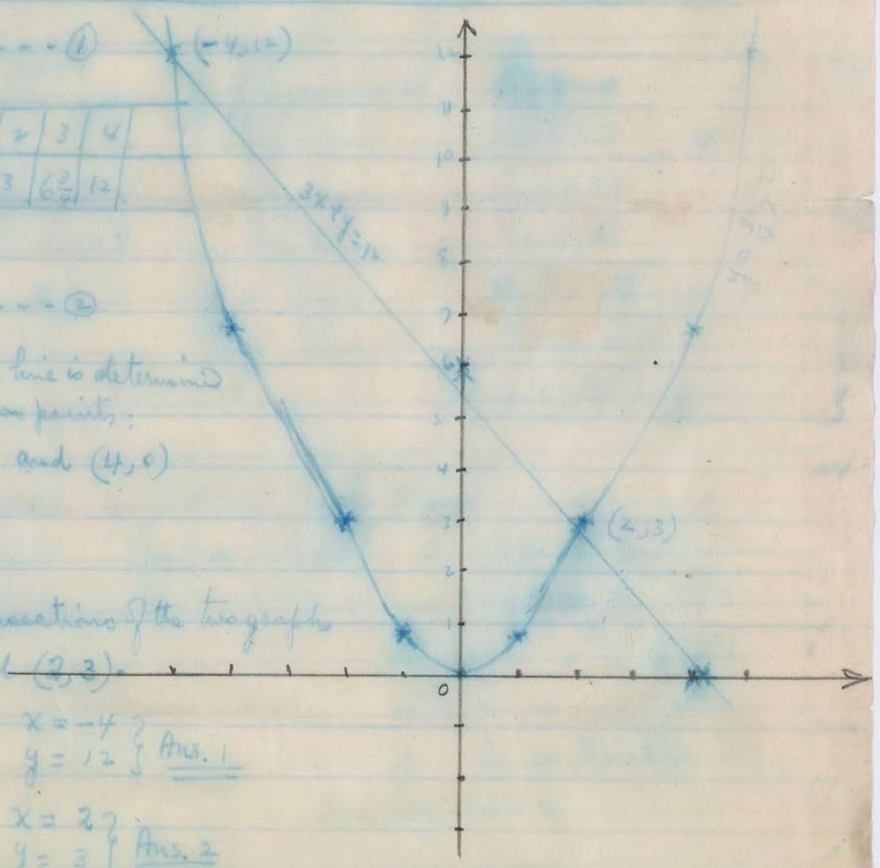
5. (i) $y = \frac{3}{4}x^2$ ----- ①

x	-4	-3	-2	-1	0	1	2	3	4
y	12	$6\frac{3}{4}$	3	$\frac{3}{4}$	0	$\frac{3}{4}$	3	$6\frac{3}{4}$	12

(ii) $3x + 2y = 12$ ----- ②

x	0	4
y	6	0

the st. line is determined by the two points: (0, 6) and (4, 0)



(iii) The points of intersections of the two graphs are: (-4, 12) and (2, 3).

∴ the solutions are: $x = -4$, $y = 12$ } Ans. 1

and $x = 2$, $y = 3$ } Ans. 2

To verify algebraically, from equation ② $y = \frac{12 - 3x}{2}$, substitute

$\frac{12 - 3x}{2} = \frac{3}{4}x^2$ ∴ $24 - 6x = 3x^2$ ∴ $3x^2 + 6x - 24 = 0$

$x^2 + 2x - 8 = 0$

$15(x^2-y^2)=7xy \quad \therefore 15x^2-7xy-15y^2=0 \quad \therefore (5x-3y)(3x+5y)=0$

$\therefore 5x=3y \quad \therefore \boxed{\frac{x}{y}=\frac{3}{5}}$ Ans. or $3x+5y=0$ or $\frac{x}{y}=-\frac{5}{3}$

or dividing $15(x^2-y^2)=7xy$ by y^2 and we get $15\left[\left(\frac{x}{y}\right)^2-1\right]=7\left(\frac{x}{y}\right)$

or $15\left(\frac{x}{y}\right)^2-7\left(\frac{x}{y}\right)-15=0 \quad \therefore \left(\frac{x}{y}-3\right)\left(5\frac{x}{y}+5\right)=0 \quad \therefore \frac{x}{y}=\frac{3}{5}$

or $\frac{x}{y}=-\frac{5}{3}$ to be discarded

2. (i) $\sqrt{\frac{(0.004678)^2\times 1.002}{(30.04)^3}}=x$

$\log 0.004678 = \bar{3}.6700$
$\log 1.002 = 0.0008$
$\log 30.04 = 1.4777$

$2\log 0.004678 = \bar{5}.3400$
$\log 1.002 = 0.0008$
$\log \text{Num.} = \bar{5}.3408$
$\log \text{Den.} = 4.4331$
$= \overline{10.9077}$
$2\log x = \bar{2}.18154$
$\log x = \bar{1}.09077$
$x = 0.01519$ Ans.

(ii) $\log 70 = 1.8451$
$\log 110 = 2.0414$
$\log 34.62 = 1.5394$

$\sqrt[3]{41503} = \sqrt[3]{7^3\times 11^2} = x$

$\log 7 = 0.8451 \quad \therefore 3\log 7 = 2.5353$
$\log 11 = 1.0414 \quad \therefore 2\log 11 = 2.0828$
$\therefore 3\log x = 4.6181$
$\therefore \log x = 1.5394$
$\therefore x = 34.62$ Ans.
Since $\log 34.62 = 1.5394$

3. In the 11 lbs of the first alloy there are 6 lbs of metal A + 5 lbs of B
In x lbs of the second alloy there are $\left(\frac{7}{7+13}x\right)$ lbs of metal A + $\left(\frac{13}{20}x\right)$ lbs of B
$\therefore \left(6+\frac{7}{20}x\right)$ lbs $= \frac{40}{100}(11+x) \quad \therefore 6+\frac{7x}{20}=\frac{2}{5}(11+x)$ or $120+7x=88+8x$
$\therefore x = 32$ lbs. Ans.

SHAMASH SECONDARY SCHOOL

Final Examination, 1958-1959.

Subject : Algebra
Class : 4th Year Secondary

Date : 28/5/1959
Time : 8:00-10:30 a.m.

All questions are to be attempted.

1. (a) Resolve into factors: (i) $(7x + 8)^2 - 2(7x + 8) - 15$. (3 marks)

(ii) $2(x - y)^2 - 3x + 3y - 5$. (4 marks)

(iii) $a(a - 4) - b(b - 4)$. (3 marks)

(b) If $15(2x^2 - y^2) = 7xy$, and if x and y are both positive, find the ratio of x to y. Use the shortest possible way. (10 marks)

2(i) Using tables, compute by logarithms the value of :

$$\sqrt[5]{\frac{(0.004678)^2 \times 1.002}{(30.04)^3}}$$ (10 marks).

(ii) Given : log70 = 1.8451, log110 = 2.0414, log34.62 = 1.5394,

compute, without using tables, the value of : $\sqrt[3]{41503}$, correct to four sugnificant figures. (10 marks).

3. A certain alloy contains 6 parts by weight of a metal A and 5 parts by weight of a metal B; another alloy contains 7 parts by weight of A and 13 parts by weight of B. If these alloys are melted and mixed together, how many pounds of the second alloy must be mixed with 11 pounds of the first alloy to make a mixture which contains 40 per cent. of A ? (20 marks).

4. (a) The expression $2 - \frac{2^{n+1}}{3^n}$ is a formula for the sum of 'n' terms of a certain geometric series, n being any positive integer. Find the first term of the series, the common ratio, and the formula for the n-th term. (10 marks).

(b) The first and second terms of a series are 'a' and 'b' respectively. Find the n th term (i) if the series is an arithmetic series; (ii) if it is a geometric one. (10 marks).

5. (i) Taking $\frac{1}{2}$ in. as one unit on the x-axis and on the y-axis, plot the curve $y = \frac{3}{4}x^2$ for values of x between $x = -4$ and $x = 4$. (7 marks).

(ii) On the same axes of coordinates draw the graph of the equation $3x + 2y = 12$. (6 marks).

(iii) From the above graphs find two roots for the simultaneous equations $y = \frac{3}{4}x^2$ and $3x + 2y = 12$. Verify your graphical answers by solving algebraically. (7 marks).

Shamash Seendary School

Final Examination, 1955–1956

Subject: Algebra — Date: 30/5/56

Class : 4th Year — Time : 8:00 – 10:00 a.m.

All questions are to be attempted:

1. In 1955 a housewife could buy 7 more eggs for 10s. 6d. than she can buy for 14s. in 1956, when the price per egg has increased by one penny. Find the price of eggs, per dozen, in 1955. (Ans. 2s.)

2. A motorist travels a certain distance x at a certain uniform speed. If his speed had been 4 mils per hour greater, he would have saved 10 minutes on the journey; and if his speed had been 9 mils per hour greater he would have saved 20 minutes. Find the distance x. (Ans. 60 miles)

3. Using one pair of axes draw graphs of x^2 and of $\frac{1}{2}x+2$ between the values of $x=-3$ and $x=+3$, choosing your own scales. From your graphs read off the solutions of $x^2=\frac{1}{2}x+2$. By drawing a further graph find the solutions of $x^2=\frac{1}{2}x+1$. (Ans. 1.7 or −1.2, 1.3 or −0.8).

4. (i) Compute by logarithms $\sqrt[7]{\frac{(0.00092)^2\times(4.006)^3}{(0.00624)^5}}$

(ii) Find the value of x from the equation $4^{2-x}\times 3^{x}=243$

5. (i) The sum of the first 29 terms of an Arithmetical progression, whose common difference is (−0.8), is zero. Find the first term. (Ans. 11.2)

(ii) Find the tenth term + the sum of the first ten terms of the Arithmetical progression whose n^{th} term is $3-\frac{1}{2}n$ (Ans. −2, 2.5).

[illegible] Examination, September 1954

Algebra

[illegible] 4th (Scientific)

Date: 21st Sept., 1954

Time: 8:30 — 10:30 a.m.

All questions are to be attempted.

1. Find by how much $(x+a)^2 + (x-a)^2$ exceeds twice x^2. Hence find the difference between $(4.3)^2 + (4.1)^2$ and $2(4.2)^2$.

2. Solve the equations: (i) $3x^2 + 1.7x - 2.6 = 0$ (correct to two decimal places)

(ii) $4x^2 - 8xy + 4y^2 - 3x + 3y - 1 = 0$; $x + 2y = 7$

3. Factorise: (i) $6x^2 + 7x - 20$

(ii) $(p^2 + pq + q^2)^2 - (p^2 - pq + q^2)^2$

(iii) $x^3 - x^2 + 2x - 1$

4. (i) What kind of series is $\frac{1}{4}, \frac{3}{10}, \frac{7}{20}, \frac{2}{5}, \ldots$? Find its n^{th} term and the sum of the first ten terms.

(ii) Three numbers in A.P. add up to 36. When they are increased by 1, 4, 43 respectively they form a G.P. What are the numbers?

5. A manufacturer produces a motor car at a cost of £440. He sells it to a dealer at a loss, and the dealer sells it to a customer for £480. Given that the dealer's percentage profit is double the manufacturer's percentage loss, find at what price the manufacturer sold the car to the dealer.

6. Draw the graph of $y = 6 + 3x - x^2$ for values of x from -2 to 5, taking 1 in. as unit on the x-axis and $\frac{1}{2}$ in. as unit on the y-axis.

From your graph find (i) the maximum value of y (correct to one dec. pl.) and (ii) between what values of x the function is positive.

(ii) $\frac{6\sqrt{x}-7}{\sqrt{x}-1} - 5 = \frac{7\sqrt{x}-26}{7\sqrt{x}-21}$ $\therefore (6\sqrt{x}-7)(7\sqrt{x}-21) - 5(\sqrt{x}-1)(7\sqrt{x}-21) = (7\sqrt{x}-26)(\sqrt{x}-1)$

$\therefore 42x - 175\sqrt{x} + 147 - (35x - 140\sqrt{x} + 105) = 7x - 33\sqrt{x} + 26$

$24\sqrt{x} = 16$ $\therefore \sqrt{x} = 8$ $\therefore x = 64$ Ans.

2.(i) $\frac{\sqrt[3]{1.002}\ \sqrt[7]{(0.005001)^2}}{\sqrt[4]{(0.03)^3 \times (4.003)^5}} = x$

$\log 1.002 = 0.0003$
$\log 0.005001 = \bar{3}.6981$
$\log 0.03 = \bar{2}.4771$
$\log 4.003 = 0.6024$

$2\log 0.005001$	$= \bar{5}.3962$
$3\log 0.03$	$= \bar{5}.4313$
$5\log 4.003$	$= 3.0120$
$3\log 0.03$	$= \bar{5}.4313$
$5\log 4.003$	$= 3.0120$
	$\bar{3}.4433$
$\log$ Den.	$= T.85848$

$\frac{1}{3}\log 1.002 = 0.0003$
$\frac{2}{7}\log 0.005001 = T.3426$
$\log$ Num. $= T.3429$
$\log$ Den. $= T.85848$
$\therefore x = \bar{1}.48444$
$\therefore x = 0.3051$ Ans.

(ii) $2^x = 8^{y+1}$; $9^y = 3^{x-3}$ $\therefore x\log 2 = (y+1)\log 2^3$; $2^x = (2^3)^{y+1}$; $(3^2)^y = 3^{x-3}$

$\therefore 2^x = 2^{3y+3}$ and $3^{2y} = 3^{x-3}$ $\therefore x = 3y+3$ and $2y = x-3$

or $\left.\begin{matrix} x-3y=3 \\ x-2y=9 \end{matrix}\right\}$ $\therefore \left.\begin{matrix} y=6 \\ x=21 \end{matrix}\right\}$ Ans.

3.(i) $\left.\begin{matrix} t_3 = 15 \\ t_{18} = 90 \end{matrix}\right|$ $\left.\begin{matrix} t_{100} = ? \\ S_{100} = ? \end{matrix}\right|$ $\left.\begin{matrix} 15 = a+2d \\ 90 = a+17d \end{matrix}\right\}$ $\therefore 75 = 15d$ $\therefore d = 5$ $\therefore a = 15-10 = 5$

$\therefore a = 5,\ d = 5$

$t_{100} = a+99d = 5+99\times5 = 500$ Ans.1

$S_{100} = \frac{n}{2}(a+l) = \frac{100}{2}(5+500) = 505\times50 = 25250$ Ans.2

(ii) $(1+x+x^2+\dots+x^{n-1})(1-x+x^2-x^3+\dots+x^{n-1}) = 1+x^2+x^4+\dots+x^{2n-2}$

$S_1 = \frac{1(x^n-1)}{x-1}$, $S_2 = \frac{1[1-(-x)^n]}{1+x}$; $S_3 = \frac{1[(x^2)^n-1]}{x^2-1} = \frac{x^{2n}-1}{x^2-1}$

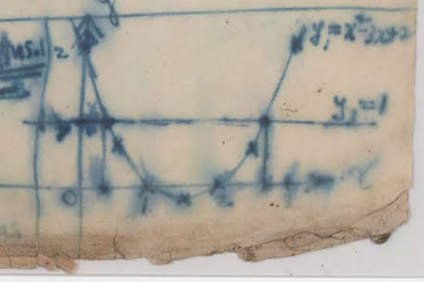
$S_1 \times S_2 = \frac{x^n-1}{x-1} \times \frac{x^n+1}{x+1} = \frac{x^{2n}-1}{x^2-1} = S_3$ Q.E.D.

4. $x^2-3x+2 = y_1$; $y_2 = x^2-3x+1$; $y_3 = 1$ $\left.\begin{matrix} x=2 \\ x=1 \end{matrix}\right\}$ Ans.

x	y_2
0	2
1	3
2	1
3	1
4	5

If we subtract y_3 from y_1, we get y_2

∴ the solution of x^2-3x+1 amounts to solving y_1 and y_3 simultaneously; when we get: $x = 2, 1$



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مدرسة فرنك عيني
متوسطة وابتدائية
بغداد
رقم التلفون ٩١٦٩٣

No.................... الرقم.......................

Date.................................. Final Exam. in Algebra Cont. التاريخ.......................

5. (i) $y = 3\cos 60°$, $\therefore AC = 4 + y = 4 + 3\cos 60°$

$z = 3\sin 60°$, $\tan\alpha = \frac{z}{AC} = \frac{3\sin 60°}{4+3\cos 60°}$

$\therefore \tan\alpha = \frac{\frac{3\sqrt{3}}{2}}{4+\frac{3}{2}} = \frac{3\sqrt{3}}{8+3} = \frac{3\sqrt{3}}{11} = \frac{3 \times 1.732}{11} = \frac{5.196}{11} = 0.47236$ or

$\tan\alpha = \frac{3 \times 0.8660}{4 + 1.5000} = \frac{2.5980}{5.5} = 0.47236$

$\therefore \alpha = 25° 17'$ $\therefore \theta = 90 - \alpha = 64° 43'$ Bearing of B from A Ans.

$\frac{z}{x} = \sin\alpha$ $\therefore x = \frac{z}{\sin\alpha} = \frac{3\sin 60°}{\sin\alpha} = \frac{3 \times 0.8660}{0.4271} = \frac{2.5980}{0.4271} = 6.0828$ miles Ans.

(ii) Let each side of the equilateral triangle $= 2x$,
then its height $AD = x\sqrt{3}$

$y = x\sqrt{3}\sin 60° = \frac{x\sqrt{3} \cdot \sqrt{3}}{2} = \frac{3x}{2}$

$\sin\theta = \frac{y}{2x} = \frac{\frac{3x}{2}}{2x} = \frac{3x}{4x} = \frac{3}{4} = 0.7500$

~~$\therefore \theta = 36° 54'$~~ Ans.

$\theta = 48° 36'$ or $48° 35'$ Ans.

Illustrative Examples:

1. In the financial year 1954-55 a man had an earned income of £1350 and a further unearned income of £200 from investment. In that year income-tax was levied according to the following rules:

The first £210 of the man's income was free of tax and there was a further tax free allowance of $\frac{2}{9}$th. of the earned income. The remainder of the earned income, called "taxable income" was taxed as follows: The first £100 of the taxable income was taxed at 2s. 6d. in the £, the next £150 at 5s, the next £150 at 7s. and the remainder at 9 s. in the £. How much tax did the man pay?

solution:

Total income = £1350 + £200 = £1550,

Taxable income = £1550 − {£210 + £$\frac{1350 \times 2}{9}$} = £1040

£100 @ 2s 6d	£100 × 0.125 = £12.500
£150 @ 5s	£150 × 0.250 = £37.500
£150 @ 7s	£150 × 0.350 = £52.500
Remainder (£640)	£640 × 0.450 = £288.000
	Total tax = £390.500
	or £390 10s

Ans.

P.T.O.

SHAMASH SECONDARY SCHOOL

FINAL EXAM. MAY 1964.

Subject : Arithmetic & Trigonometry Date :

Class : 4th Year Secondary Time : 8:00 - 10:30 a.m.

Answer All Questions.

1. Two lighthouses A and B are 5 miles apart, B being due east of A. A ship at P is due north of A, and on a bearing 322° (N.38°W.) from B. The ship then sails in a direction 056° (N.56° E.) to a position Q which is due north of B. Calculate PQ, BQ, and the bearing of Q from A.

PQ = 6.031M. BQ = 9.779M N 27° 6' E

2. The elevation of a spire from a point A due N. of it is 28°, and from a point B due E. of it 18°. Find the height of the spire if AB is 100 yards.

27.7 yds

3. A city council requires an annual rate of 16s. 4d. in the £ of rateable value. Out of each 16s. 4d. received the sum of 5s. 7.3d. is spent on education.

(i) What is the rateable value of a house, the holder of which pays £ 13 1s. 4d. half-yearly in rates ? £32

(ii) If this householder has two children, both attending the council's school, what sum, to the nearest $\frac{1}{10}$ d., is he contributing per week towards the education of each child ? (Take a year to be 52 weeks.)

1s 8.7d

4. A man invests £ 30,155, partly in 5½% stock at 86 and the rest in 4½% stock at 99. He divides the money so as to obtain the same income from each stock. Find the total income.

1295

5. Find, correct to three significant figures, the weight in pounds of a cylindrical iron pipe 10 ft. long, whose outer diameter is 1 ft. 6 in. and inner diameter 1ft. 4 in., given that 1 cu. ft. of iron weights 494 lb. (Take π as 3.142.)

1830

If the inner diameter is increased to 1 ft. 5 in., the outer diameter and the length remaining unaltered, find the ratio of the new weight to the old.

$\frac{35}{66}$

PQ = 5 cosec 56° 5 cosec 56°

BQ = 5 cot 58° + BL cot 38°

##############

2(a) The total rateable value of a town is £30000. How much does the town receive in a year if the rate levied is 16s in the £,

(b) If the town estimates that it will need an extra £3750 next year, by how much will the rate in the £ have to be increased if the total rateable value remains unchanged?

(c) If the town decided to leave the rate in the £ at 16s but to get the extra £3750 by increasing the rateable value of all property, what ought to be the new rateable value of a house which was formerly rated at £32.

Solution:

(a) Total rates = £30000 × 0.8 = £24,000

(b) $\frac{3750}{30,000}$ = £0.125 = 2s 6d Increase in rate

New Rate = 16s + 2s 6d = 18s 6d

(c) $\frac{\text{New Rateable Value}}{\text{Old Rateable Value}} = \frac{\text{New tax (Rates)}}{\text{Old tax (Rates)}}$ (provided the rate in the £1 remains the same)

$\frac{\text{New Rateable Value of the House}}{\text{Old " " " "}} = \frac{27,750}{24000}$

New R.V. = $\frac{\text{old R.V.} \times 27,750}{24,000}$

Solution:

(i)

Yearly rates = (£13 1s 4d) × 2

= £26 2s 8d

~~Ratable~~

Rateable value of the House = $\frac{£26\ 2s\ 8d}{16s\ 4d}$

$= \frac{522\frac{2}{3}}{16\frac{1}{3}}$

$= \frac{1568}{49} =$ ~~21~~ £32

(ii) (£13 1s 4d) $\frac{5s\ 7.3d}{16s\ 4d}$

$= \frac{3136 \times 67.3}{196} = 16 \times 67.3$

$= 1076.8$ d/year

$\frac{1076.8}{52} = 20.7$ d

or 1s 8.7d per week per child

New R.V = $\frac{32 \times 27,750}{24,000}$

= £37 Ans.

Stencil

Taxation

(a) Income tax ضريبة الدخل

(b) Rates = tax on ~~rateable~~ real property ضريبة الأملاك

(a) Income tax

Income tax is levied in accordance with ascending scale (تصاعدية) i.e. the rate of tax increases with higher income

Earned income = Income ~~as a~~ resulting from one's toil (العمل)
e.g. salaries

Unearned income = Income from deposits, shares etc.

In the U.K income tax is ~~calculated~~ levied at the rate of so many shillings in the £ e.g 10s 6d of £1 of income.

Total income = Total earnings

Taxable income = Total income - tax-free allowance

— — —

(b) Rates

Rates – ضريبة الأملاك

Rent – الإيجار

Rateable value – assessed value
قيمة التقدير

The rates are collected on

P.T.O

the rateable value which may or may not be equal to the rent. Rates are ~~d~~ levied on the basis of so many shillings in the £1 of rateable value

e.g the rate of a house is 5s in the £

If the rateable value is £100, the rates are £25.

The rates may sometimes exceed the rateable value; Reason: the assessment was made many years ago when rents were low. Instead of making reassessment, it is sometimes decided to raise the rate or the tax per £1. We thus find in certain boroughs the rate is 25s or more in the £.

Penny rate = a rate of 1d in the £1 of rateable value.

(a) Ans

1 (a) Amount of stock obtained =

$$\frac{£2286}{95\frac{1}{4}} \times 100 = £2,400$$

Dividend = $\frac{2400}{100} \times 4\frac{1}{2} = £108$

Amount realized from sale of stock = $\frac{£2400}{100} \times 98$

= £2,352

Total gain = £2,352 + £108 − £2,286

= £174

(b) I must set out at 10.25 − 00.15 = 10.10 a.m (true time)

There are 100 minutes between 8.30 a.m and 10.10 a.m.

My watch gains in 100 minutes of true time $\frac{6}{60} \times 100 = 10$ minutes

Latest time indicated by watch at which I must set out is 10.20 a.m

2 (a)

Speed in ft/sec

$$\frac{1000}{6\frac{1}{4}\times 60} \div \left(\frac{22}{7}\times\frac{1}{32}\right)$$

$$= \frac{1000\times 4\times 7\times 9}{25\times 60\times 22} = \frac{84}{11} = 7\frac{7}{11} \text{ ft/sec}$$

(b) Sediment

$$\frac{1000\times 4}{25}\times 60\times 48\times\frac{1}{2}\times\frac{1}{16}\times\frac{1}{2240} = \frac{45}{7}$$

$$= 6\frac{3}{7} \text{ tons}$$

3 (i)

Merchant's total Cost.

15 X 18 + (6s 3d) 4.6 + £5 10s =

£270 + £1 8s 9d + £5 10s = £276 18s 9d

(ii) Total amount merchant received from sale

$$92 \times (£1\ 5s) + \frac{15 \times 112 \times 4}{240} + \{15 - (4.6 + 0.75)\}20$$

= £115 + £28 + £193 = £336

(iii) Profit 336 − 276 18s 9d = £59 1s 3d

$$\text{Percentage profit} = \frac{59.0625 \times 100}{276.9375}$$

$$= \frac{5906250\,0}{2769375}$$

Since to two significant figures we need only take four in calculations

$$\text{Percentage profit} = \frac{5906\,0000}{2769\,000}$$

= 21 %

4) $52,320 \times 0.6375 + 127,460 \times 0.9625 =$

£33,354 + £122,680 5s =

£156,034.25

Average rate =

$$\frac{£156,034.25}{£52,320 + £127,460}$$

$$= \frac{£156,034.25}{£179,780}$$

= £0.8679

= £17s $4\frac{1}{4}$d to nearest farthing

Shamash Secondary School
Final Examination, May, 1967.

Subject: ~~Arithmetic~~ & Trigonometry
Class: 4th Year Secondary.

Date: ~~17~~/5/1967
Time: 8:00 - 10:30 a.m.

Answer five questions which must include questions 2 & ~~5~~ 3.

1. (a) How much stock is obtained by investing £2,286 in a 4½ per cent stock at 95¼? After receiving the first annual dividend on this stock, it is immediately resold at 98. Calculate the total gain on the transaction.

   (b) I have a watch which gains six minutes in every true hour. I put the watch right at 8.30 a.m. What is the latest time indicated by the watch at which I must set out to catch a train which leaves at 10.25 a.m. if it takes me 15 minutes to walk to the station ?

2. Following a storm, water is pumped out of a flooded area through a pipe of 8 in. diameter at the rate of 1,000 gallons per minute. Taking 1 cu.ft. as 6¼ gallons and π as $\frac{22}{7}$, calculate:

   (a) the speed in ft. per sec. at which the water is passing through the pipe.

   (b) how many tons of sediment will be pumped out in two days if it is known that the flood water contains ½ oz. of sediment in every cu.ft. of water.

3. A merchant bought 15 tons of potatoes from a farmer at £18 per ton.

   (a) He sold 4 ton 12 cwt of the potatoes in 1 cwt bags at £1 5s. per bag. The additional cost to the merchant in selling the potatoes in this way was 6s. 3d. per ton.

   (b) He sold 15 cwt retail at 4d. per lb for which he incurred additional labour costs of £5 10s.

   (c) He sold the remainder of the potatoes in bulk at £20 per ton. Calculate:

      (i) the merchant's total costs, including the initial cost of the potatoes, cost of selling the potatoes in bags, and additional labour cost for the retail sales.

      (ii) the total amount the merchant received from his sales.

      (iii) the merchant's profit calculated as a percentage, correct to 2 significant figures, of his total costs.

4. A borough is divided into two districts whose rateable values are respectively £52,320 and £127,460. The rate in the first district is 12s. 9d. in the £, and in the second district it is 19s. 3d. in the £. Find the average rate for the whole borough to the nearest farthing.

(cont'd.p.2)..

Arith. & Trig. 17/5/67
4th Year Secondary.

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5. ABC is a triangle with AB = AC = 100 ft. and the angle BAC is 70°. At A is a vertical pole AO = 80 ft. high. Calculate:

(i) the length of BC,

(ii) the angle of elevation of the top of the pole from B,

(iii) the angle of elevation of the top of the pole from the mid-point of BC.

6. A, B and C are three points on a coastline which runs from north to south; B is south of A and C is 1000 yards south of B. A boat is moving in a straight line towards C and when it is at a point P which is 2000 yd. from A on a bearing of (N.60.E.) its bearing from B is (N.38E.). Calculate:

(a) the distance from P to the nearest point X on the coastline.

(b) the distance AB.

(c) the bearing of P from C.

Shamash Secondary School
Final Examination, May, 1967.

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Arith. & Trig. 17/5/67
4th Year Secondary.

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Shamash Secondary School
Final Examination, May, 1967.

Subject: Arithmetic & Trigonometry — Date: 17/5/1967
Class: 4th Year Secondary. — Time: 8:00 - 10:30 a.m.

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(cont'd.p.2)..

Arith. & Trig. 17/5/67
4th Year Secondary.

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(b) the distance AB.

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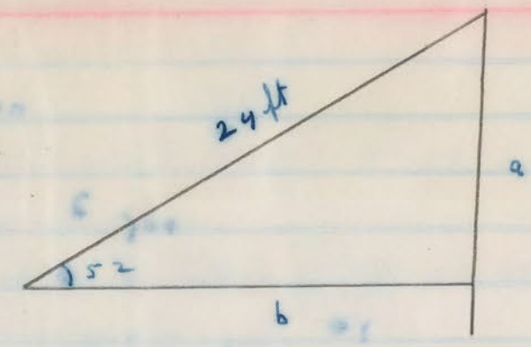
Solution to Arithmetic + Trig Exam:
Mid-year Exam.; Jan, 29th 1961
4th Year.

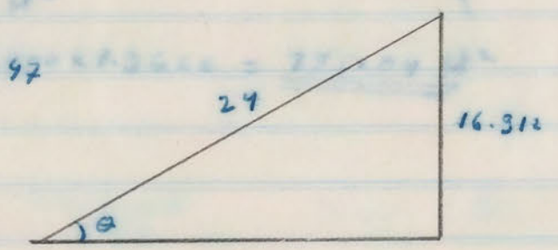


a: $24 \sin 52 = 24 \times 0.7880 = 18.912$ ft

b: $24 \cos 52 = 24 \times 0.6157 = 14.7768$ ft.

$18.912 - 2 = 16.912$ ft

$\sin\theta = \frac{16.912}{24} = 0.7047$



$\therefore \theta = 44^\circ\ 48'$

b': $24 \cos 44^\circ\ 48'$

$= 24 \times 0.7096 = 17.0304$

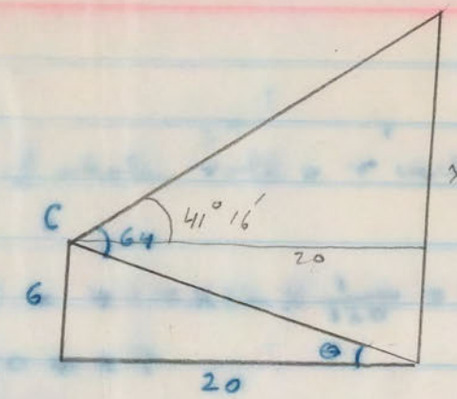
$17.0304 - 14.7768 = 2.2536$ ft

2.25 ft.

$\tan\theta = \frac{6}{20} = 0.3000$

$\therefore \theta = 16^\circ\ 42'$

$63^\circ\ 60'$
$-\ 16^\circ\ 42'$
$47^\circ\ 18'$



$x = 20 \tan 41^\circ 18'$

$= 20 \times 0.8920 = 19.84\ ft$ $\quad 20 \times 1.0837$

$19.84 + 6 = 25.84\ ft$ $\quad 21.674$
$\quad 6$
$\quad 27.674$

(ii)

$8 \times 50 = 400\ ft^2$

$400 \cot 27 = 400 \times 1.9626 = 785.04\ ft^2$

4th year, ~~February~~ Jan., 29th 1967

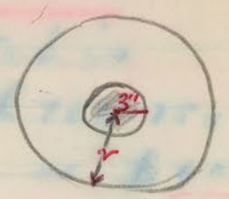
3

(a) $\frac{432,000}{540,000} = £0.8$ or 16 d

(b) £63 × 0.8 = £50.4 or £50 8 d.

(c) $\frac{540,000}{240} =$ £2250.

4



Let radius of whole roll = r inches

$\pi r^2 - 9\pi = 4800 \times 12 \times \frac{1}{120} = 480$

$\pi r^2 = 480 + 9\pi$

$r^2 = \frac{480}{\pi} + 9$

$= \frac{480}{3.1416} + 9$

$= 161.79$

$r = 12.72$ inches.

5) $\frac{990 \times 20}{27\frac{1}{2}} = \frac{990 \times 20 \times 2}{55} = 720$ (£1) shares

720 × 32 d = £720 × 1.6 = £1,152 he realized from the sale

$\frac{1152 \times 20}{9} = 2560$ (10 d) shares he bought

Change of income:

Income from (£1) shares = $\frac{720 \times 10}{100} =$ £72

" " (10 d) " = $\frac{2560}{2} \times \frac{5}{100} =$ £64

Income decreased by £72 − £64 = £8

6) 6000 : 4000 = 3 : 2.

At end of 1st year B's profit = $\frac{1400}{5} \times 2 = £560$

At beginning of 2nd year B's capital = £4,560

At end " " " B's profit = $\frac{1,584 \times 4,560}{10,560} = £684$

At beginning of 3rd year B's capital = £4,560 + £684 = £5,244

At the end of 3rd year B's profit = $\frac{(1639 - 1452)\ 5,244}{11,244} = £164.75$

At the end of 3rd year B's capital is

£5,244 + £764.75 = £6008.75

or £6008 15s.

Shamash Secondary School
Mid-Year Exam. January, 1967

Subject: Arith. & Trig. Date: Jan. 29,1967
Class: 4th Year Secondary. Time: 8:30 - 10:30 a.m.

Answer five questions which must include question 2.

1. A ladder, 24 ft. long, makes an angle of 52° with the ground and leans against a vertical wall. If the top of the ladder slips down 2 ft., how far will the foot of the ladder move ?

2. (i) The point C is 6 ft. above level ground and 20 ft. measured horizontally from a vertical pole AB where B is at ground level. If the angle ACB = 64°, calculate the length of AB.

   (ii) A vertical wall of length 50 ft. and height 8 ft. runs due N and S. Find the area of the shadow of the wall cast on level ground by the sun shining from the W at an elevation of 27°.

3. The total rateable value of a town is £540,000 and it is estimated that the necessary expenditure for 1959 will amount to £432,000. Calculate:

   (a) The rate in the £ which must be charged to meet the 1959 expenditure.

   (b) The rates to be paid on a house whose rateable value is £63.

   (c) The amount produced by a penny rate.

4. A length of 4800 ft. of paper is wrapped on a wooden cylinder of radius 3 in.; the thickness of the paper is $\frac{1}{120}$ in. Find the radius of the whole roll to $\frac{1}{100}$ in.

5. A man invested £990 in (£1) shares, paying 10%, at 27s. 6d.; he sold the shares at 32s. and invested the proceeds in (10s.) shares, paying 5%, at 9s. How many (10s.) shares did he buy and what was the change of his annual income ?

6. Two partners A, B started with capitals of £6000, £4000 respectively. The profits at the end of each year are divided in proportion to their capitals invested in the business at the beginning of the year. A withdrew his profits at the end of each year, while B left his in the business. The profits for the first 3 years were £1400, £1584, £1639 15s. respectively. What was B's capital at the end of 3 years?

Let x be the amount invested in £.

$2\frac{3}{4}\%$ Stock purchased = £ $\frac{x}{95} \times 100$.

$3\frac{1}{2}\%$ Stock purchased = £ $\left(\frac{100x}{95} - 900\right)$.

Since the income remained the same

$$\frac{100x}{95} \times \frac{23/4}{100} = \left(\frac{100x}{95} - 900\right)\frac{3\frac{1}{2}}{100}$$

$$\frac{11x}{1 \times 95} = \left(\frac{x}{95} + \frac{855}{95}\right)\frac{7}{2} = \frac{7x}{2 \times 95} - \frac{7 \times 855}{2 \times 95}$$

$$\frac{11x}{1 \times 95} - \frac{7x}{2 \times 95} = \frac{7 \times 855}{2 \times 95}$$

$$\frac{4x}{95} = \frac{7 \times 855}{2 \times 95}$$

$$4x = 7 \times 855$$

$$1.5x = 7 \times 855$$

$$x = \frac{7 \times 855 \times 2}{8} = £3990$$

285
14
1140
285
3990

(a)

(i) Area of surface of sphere $= 4\pi r^2$.

(ii) Volume of sphere $= \frac{4}{3}\pi r^3$

(iii) Area of curved surface of a cone $= \pi r l$

(iv) Volume of a cone $= \frac{1}{3}\pi r^2 h$

(v) Area of trapezium $= \frac{(a+b)h}{2}$.

(b) Volume of bar $= 15" \times 9" \times 8"$ cubic inches

net volume (after allowing for 10% loss $= \frac{15 \times 9 \times 8 \times 9}{10}$ cubic inches

Volume of a sphere $= \frac{4}{3} \times 3.142 \times \frac{27}{8}$

$= 1.571 \times 9$ cubic inches.

No. of spheres $= \frac{15 \times 9 \times 8 \times 9}{10 \times 9 \times 1.571} = 68$ spheres

Wt. of 68 spheres $= \frac{320}{1728} \times 68 \times 1.571 \times 9$

$= \frac{205 \times 17 \times 1.571}{12}$

$= 5474.935$ lb

$= 456$ lb to nearest lb.

205
17
1435
205
3485
1.571
3485
24395
17425
3485
5474.935

456.2
12 | 5474.935
67
74
29

Between 4 p.m. Friday and noon the following Wednesday = $4\frac{5}{6}$ days.

The fast clock gains $3 \times 4\frac{5}{6} = 14\frac{1}{2}$ minutes

The slow clock loses $2\frac{1}{2} \times 4\frac{5}{6} = 12\frac{1}{12}$

Difference between them $= 14\frac{1}{2} + 12\frac{1}{12}$

$= 26\frac{7}{12}$ minutes

Difference per day $= 5\frac{1}{2}$ minutes

The two clocks will show the same time again when the difference between them has become 12 hours. $12 \times 60 = 720$ min

$\frac{720}{5\frac{1}{2}} = \frac{720 \times 2}{11} = 130\frac{10}{11}$ days.

The fast clock gains 3 minutes per day

It will gain $3 \times 130\frac{10}{11}$ minutes in $130\frac{10}{11}$ days

$= \frac{4320}{11}$ minutes

$= 392\frac{8}{11}$ minutes

$= 6$ hours and $32\frac{8}{11}$ minutes

Now the correct clock would read after $130\frac{10}{11}$ days of correct time

1 hr $49\frac{1}{11}$ min p.m

The fast clock will therefore read

1 hr $49\frac{1}{11}$ min. + 6 hours $32\frac{8}{11}$ min

$= 8$ hours $21\frac{9}{11}$ minutes

p.m

The slow clock will read the same.

(3)

21 s 10 d + 9 s = 30 s 10 d or $30\frac{5}{6}$ s = $\frac{185}{6}$ s

Cost per year = £ $\frac{44 \times 185}{6 \times 20}$

Average cost per week = £ $\frac{11}{\cancel{44}} \times \frac{37}{\cancel{185}} \times \frac{1}{6} \times \frac{1}{\cancel{6}} \times 52 = \frac{11 \times 37}{6 \times 52} = 1.3045$

= £ 1 6 s 1 d.

Income from investment = £ $\frac{3300 \times 2.5}{100}$ = £ 82.5

Garage rent = $\frac{7.5 \times 52}{20}$ = £ 19.5.

Total expenditure = £ 126 + £ 19.5 = £ 145.5

£ 145.5 − £ 82.5 = £ 63 net expenditure

He saves £ $\frac{44 \times 185}{6 \times 20}$ = £ $6\frac{1}{3}$

= £ $6\frac{5}{6}$ − £ 63 = £ $4\frac{5}{6}$

= £ 4 $\frac{100}{6}$ s

= £ 4 $16\frac{2}{3}$ s

= £ 4 16 s 8 d

There are two clocks, one of which gains 3 minutes, while the other loses $2\frac{1}{2}$ minutes each day. They are set right at 4 o'clock on Friday afternoon. What is the difference between their readings on the following Wednesday? In how many days from the time they are set right will they both show the correct time? What will be that time?

Two clocks are set right simultaneously at 12 noon, one of which loses 6 sec. per hour, and the other gains 3 sec. in 50 min. (a) How long will it be before the minute hands are again in the same direction? (b) What will then be the time as shown by each clock? (c) When will both clocks simultaneously indicate correct time?

SHAMASH SECONDARY SCHOOL

Final Examination, May 1966

Subject: Arithmetic & Trigonometry
Class : 4th Year Secondary

Date: 18.5.1966
Time: 8:00-10:30 a.m.

- - -

Attempt five questions only including question (4).

1. A person, having bought a certain amount of $2\frac{3}{4}\%$ stock at 95, afterwards sold it, and with the proceeds bought $3\frac{1}{2}\%$ stock. He obtained £900 less stock than before, but his income was unchanged. How much money did he originally invest?

(20 marks)

2. A house holder owns his house which has a rateable value of £44 on which the annual rates are charged at 21s 10d in the £1. He also has to pay an annual property tax at the rate of 9s in the £ on an assessment of £44. Calculate, correct to the nearest penny, the average cost per week of the total of these charges, taking a year as 52 weeks. He subsequently sells his house for £3300, which sum he invests at the rate of $2\frac{1}{2}\%$ per annum free of tax, and moves into a flat which he rents at £126 per annum. He has however to rent a garage for his car at 7s 6d per week. Find how much per annum he saves by the change.

(20 marks)

3. (a) A watch was 5 minutes fast at 9 a.m. on Monday, and 10 minutes slow at 12 noon on the following Wednesday. Find when it was exactly right, assuming that it lost time uniformly.
Note:(9 a.m. and 12 noon are correct time). (10 marks)

(b) Two clocks sound the first stroke of 12 o'clock at the same instant; one clock allows an interval of 20 secs. between each stroke and the next, and the other allows 25 secs. How many strokes of the slower clock remain after the quicker one has finished striking, and what time will elapse between the 12th stroke of the quicker one and the following stroke of the slower one?

(10 marks)

4. (a) A solid consists of a hemisphere, radius 8 cm., joined to a cone of the same base-radius and height 6 cm., so that the plane surfaces coincide. Find (i) the volume, (ii) the total area of the surface of the solid. (Give answer to 3 significant figures).

(10 marks)

(b) A sphere of radius 3 in. is filed down into the greatest possible cube; find the volume of the material removed. (Give answer to 4 significant figures).

(10 marks)

5. Find the difference between the perimeters of a regular pentagon and a regular hexagon, each of which has an area of 24 square inches.

(20 marks)

6. In response to an S O S call from a ship at A, another ship at B, 175 miles due east of A, starts toward A at a speed of 12 miles per hour. At the same time a third ship at C, which is 186 miles from B in a direction bearing 5° 15′ west of north, also starts for A at a speed of 16 miles per hour. Which ship will reach A first, and how long will it take ?

(20 marks)

7265
48
58120
29060
3.4 8720

0.5-
0.5774
16
34644
5774
9.23 84

SHAMASH SECONDARY SCHOOL

Final Examination, May 1966

Subject: Arithmetic & Trigonometry
Class : 4th Year Secondary

Date: 18.5.1966
Time: 8:00-10:30 a.m.

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(20 marks)

SHAMASH SECONDARY SCHOOL

Final Examination, May 1966

Subject: Arithmetic & Trigonometry
Class : 4th Year Secondary

Date: 18.5.1966
Time: 8:00-10:30 a.m.

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SHAMASH SECONDARY SCHOOL

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(20 marks)

Shamash Secondary
Mid-Year Exam. 1965-1966

Subject: Arithmetic — Date: 1.2.1966
Class : 4th Year, Scientific — Time: 8:30 - 10:30

Answer all questions:

1. A dealer sells 2640 articles for £ 341 his profit being 24% of his outlay. Find the cost price of each article. If the cost price to the dealer increased by 8% and he does not change his selling price, find how many articles he must sell in order to obtain the same total profit as before ?

2.a) Compare the volume of the Moon with that of the Earth, if the diameter of the former be to the diameter of the latter as 27 to 100. Give your answer in the form of a ratio whose denominator is unity.

b) In order to increase the weight of a block of steel by 1 oz. a cyclindrical hole ¼ in. in diameter, is drilled in the block, and the hole is then filled with lead. To what depth must the hole be drilled if steel weighs .29 lb. per cub. in. and lead weighs .41 lb. per cub. in. ? (To nearest $\frac{1}{100}$ in.)

3. The rate in a certain town is 13s. 10d. in the £. If the rateable value is increased by 5% and the rate reduced by 6d. in the £, will the income from rates be increased or decreased, and by how much per cent ?

4. Brass is made up of copper and zinc in the proportion 5:4 by volume. If 1cc of copper weighs 8.8 gm and 1 cc of zinc weighs 7.1 gm calculate:

(a) the weight of copper required to form brass with 50 cc of zinc, 550 gm

(b) the weight of copper in 1 kilogram of brass correct to the nearest gm, 608 gm

(c) the volume of 1 kilogram of brass correct to the nearest cc. 124 cc

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ndustrial Exam in
ic Eng. thyear, 14/9/1965

come

$\left(\frac{£253\ 15s}{101\frac{1}{2}}\right) 3\frac{1}{2} = £8.75$

or £8 15s

Amount realised

$(£253\ 15s)\ \frac{99}{101.5} = £247\ 10s$

(2)

$\left(\frac{3}{2}\right)^3 = \left(\frac{3}{4}\right)^3 + 1^3 + x^3$ $\quad x$ = radius

$x^3 = \frac{27}{8} - \frac{27}{64} - 1$

$= \frac{216}{64} - \frac{27}{64} - \frac{64}{64}$

$= \frac{125}{64}$

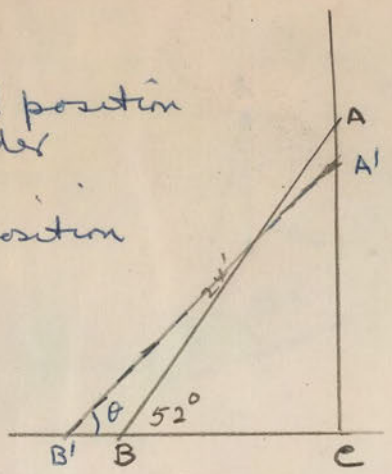
$x = \frac{5}{4}$ radius

Diam. $= 2x = \frac{10}{4} = 2\frac{1}{2}"$

(3)

B original position of ladder

A'B' later position



AA' = 2 ft
find BB'

$AC = 24 \sin 52^\circ$

$= 24 \times 0.7880 = 18.91$ ft

$A'C = 18.91 - 2 = 16.91$ ft

$\sin\theta = \frac{A'C}{A'B'} = \frac{16.91}{24} = 0.7046$

$\angle\theta = 44^\circ\ 48'$

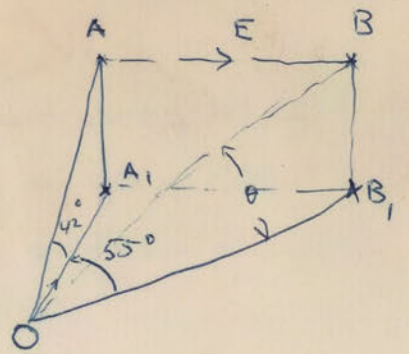
$BC = 24 \cos 52^\circ = 24 \times 0.6157$

$= 14.78$ ft.

$B'C = 24 \cos 44^\circ\ 48' = 24 \times 0.7096$

$= 17.03$ ft.

$B'B = 17.03 - 14.78 = \underline{2.25 \text{ ft}}$



$AA_1 = OA_1 \tan 42^\circ = OB_1 \tan\theta$ (a)

$OB_1 = OA_1 \sec 55^\circ$ (b)

$\tan\theta = \frac{OA_1}{OB_1} \tan 42^\circ$ from (a)

$= \frac{OA_1 \tan 42^\circ}{OA_1 \sec 55^\circ}$ from (b)

$= \tan 42^\circ \cos 55^\circ$

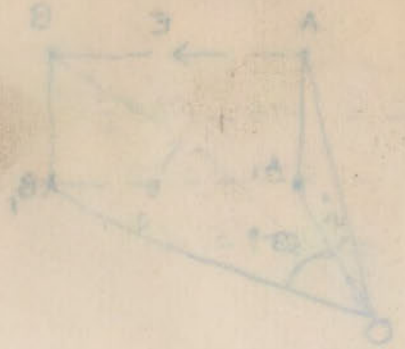
~~$= 0.9004 \times 0.5736$~~

$= 0.9004 \times 0.5736$

$= 0.5165$

θ = angle of elevation

$= 27^\circ \quad 19'$



(5)

Alternative I

$$68 \times 7 + 196 \times 2\tfrac{3}{4} = 476 + \cancel{196} \overset{49}{} \times \tfrac{11}{\cancel{4}}$$

$$= 476 + 539$$

$$= 1015\,d,$$

Alternative II

$$(68 + 196) \times 1\tfrac{1}{2}d + £2\ 5s\ 6d =$$

$$264 \times \tfrac{3}{2}\,d + 546\,d = 942\,d$$

Method II is cheaper by

$$1015 - 942 = 73\,d$$

or 6s 1d

$117 \times 7 = 819\,d$

or £3 8s 3d for light

£5 3s 1d − £3 8s 3d

= £1 14s 10d for power

No. of units for power is:

$$\{£1\ 14s\ 10d\} \div 2\tfrac{3}{4}d$$

$$= 418 \times \frac{4}{11} = 152 \text{ units}$$

for power

Shamash Secondary School
Conditional Exam. Sept.1965

Subject: Arithmetic & Trigonometry
Class: 4th Year Secondary.

Date: 14/9/1965
Time: 8.00-10.30

Attempt all questions.

(1) Find the income produced by investing £253 15s. in 3½% stock at 101½ and the amount realised by subsequently selling out at 99.

(2) A spherical ball of lead 3 in. in diameter is melted and recast into three spherical balls. The diameters of two of these are 1½ in. and 2 in. respectively. What is the diameter of the other ?

(3) A ladder, 24ft. long, makes an angle of 52° with the ground and leans against a vertical wall. If the top of the ladder slips down 2 ft. how far will the foot of the ladder move ?

(4) An aeroplane is flying horizontally due E. When it is due N. of an observer its elevation is 42°. Find its elevation when it is N. 55° E. of the observer.

(5) A householder has two alternative methods of paying for the electric light and power that he uses during one quarter of a year. Either he pays 7d. per unit for light and 2¾d. per unit for power, or he pays 1½d. per unit for light and for power and also a quarterly charge of £2 5s. 6d.

5 Determine which method is the cheaper, and by how much, for a quarter during which he uses 68 units for light and 196 units for power.

A householder paying by the first method used 117 lighting units during a quarter, and his electricity bill for the quarter was £5 3s. 1d. Find the number of units used for power.

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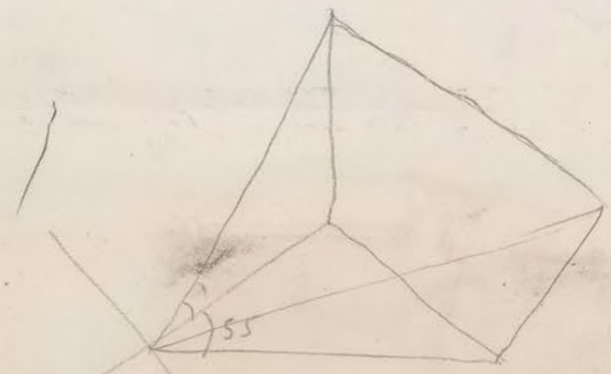
240 [illegible]



Shamash Secondary School

Subject : Arithmetic & Trigonometry — Date : 14/9/1965

Class : 4th Year Secondary — Time : 8.00-10.30

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   A householder paying by the first method used 117 lighting units during a quarter, and his electricity bill for the quarter was £5 3s. 1d. Find the number of units used for power.

1st Quarter Exam.

Subject: Arithmetic & Trigonometry. Date: 1/12/1958
Class: 4th Secondary. Time: 90 minutes.

Answer all questions.

1 (a) Decimalise: -

£ 9	15s	$10\frac{3}{4}$d
£ 5	2s	$2\frac{3}{4}$d
£10	10s	$8\frac{3}{4}$d.

(b) Express in shillings and pence to the nearest $\frac{1}{4}$d

£0.840, £0.730, £0.910

(c) Express as a compound quantity correct to the nearest unit of the lowest given denomination.

0.6186 of $7\frac{1}{2}$ tons. (tons, cwt., qr.).

2. A man walks from his house to a town 6 miles away at 4 miles per hour and cycles back again at 12 miles per hour. Find his average speed for the double journey.

3. A wireless pole stands at the corner A of a rectangular court ABCD. Its elevation from the corner B is 50^0. Find its elevation from the opposite corner C; given the length of AB = $\frac{3}{4}$ that of AD.

4. A ship steaming S60^0E at 10 miles an hour is 10 miles N of a lighthouse at 12.00 noon. At what time will the ship be due E. of the light house ?

5. The area of a rectangular field is 2 acres and its breadth is 88 yds - Find the perimeter of the field and the length of the diagonal correct to the nearest yard.

1st Quarter Exam.

Subject: Geometry
Class: 4th Secondary

Date: 30/11/1958
Time: 90 minutes.

Attempt all questions.

1. Draw a triangle given two angles and the perimeter. Prove your construction. (30 marks).

2. Construct the quadrilateral A B C D given that AB=AD= 8cm. BC=CD = 6 cm. and the angle ABC = 75^{o}.

   Construct the point X on AB produced such that triangle AXD is equal in area to the quadrilateral ABCD. Measure AX.

   (35 marks)

3. ABC is a triangle. Y & Z are the mid-points of AC and AB respectively. AC is produced to D so that CD = AY. DP is drawn parallel to BA to meet BC and ZY (both produced) at P and W respectively. Show that the triangles DCP and AYZ are congruent.

   Calculate the ratio of the area of the parallelogram ZBPW to that of the triangle ABC.

   (35 marks).

Shamash Secondary School

1st.Quarter Exam.

Subject: Arithmetic & Trigonometry Time: 12:00-1:30 p.m.
Class : 4th year Secondary. Date: 8/12/1957

All questions are to be attempted.

(1) State to how many significant digits are the following underlined numbers given ?

My expected profit from my business in the year 1960 is £ 8500. I have to pay my landlord with whom I have just concluded a 10 years agreement £ 1250 per annum. I have to pay my assistant a fixed sum of £ 500 per annum plus a commission of 0.5 per cent on my turnover. His earning from commission may amount to £ 650 per annum. My business premises measures 19.10m, by 25.00 m.

(2) (a) Decimalise to 3 places the following:

£ 2	12s	$2\frac{3}{4}$d
£ 8	10s	$8\frac{1}{2}$d
£ 9	5s	$10\frac{1}{4}$d

(b) Convert into shillings and pence to nearest $\frac{1}{4}$d the following:

£ 0.509 , £ 0.620, £ 0.945.

(c) Express 4.316 gallons into gallons, quarts and pints to the nearest pint.

(3) A watch which gains 5 sec.in every 3 min. of true time was set right at 6 a.m. What was the true time in the afternoon of the same day when the watch indicated a quarter-past 3 O'clock ?

(4) The average age of m boys is b years and of n girls is c years. Find the average age of all together.

(5) At 9 a.m. a ship which is sailing in a direction E.37^{0}S. at the rate of 8 miles an hour observes a fort in a direction 53^{0} North of East. At 11 a.m. the fort is observed to bear N.20^{0}W., find the distance of the fort from the ship at the first observation.

(6) From the roof of a house 30 feet high the angle of elevation of the top of a monument is $42^{0}7'$, and the angle of depression of its foot is 17^{0} 59'. Find its height.

Shamash Secondary School

1st. Quarter Exam.

Subject: Arithmetic & Trigonometry Time: 12:00-1:30 p.m.
Class : 4th year Secondary. Date: 8/12/1957

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Shamash Secondary School

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My expected profit from my business in the year 1960 is £ 8500. I have to pay my landlord with whom I have just concluded a 10 years agreement £ 1250 per annum. I have to pay my assistant a fixed sum of £ 500 per annum plus a commission of 0.5 per cent on my turnover. His earning from commission may amount to £ 650 per annum. My business premises measures 19.10m, by 25.00 m.

(2) (a) Decimalise to 3 places the following:

£ 2	12s	$2\frac{3}{4}$d
£ 8	10s	$8\frac{1}{2}$d
£ 9	5s	$10\frac{3}{4}$d

(b) Convert into shillings and pence to nearest $\frac{1}{4}$d the following:

£ 0.509 , £ 0.620, £ 0.945.

(c) Express 4.316 gallons into gallons, quarts and pints to the nearest pint.

(3) A watch which gains 5 sec.in every 3 min. of true time was set right at 6 a.m. What was the true time in the afternoon of the same day when the watch indicated a quarter-past 3 O'clock ?

(4) The average age of m boys is b years and of n girls is c years. Find the average age of all together.

(5) At 9 a.m. a ship which is sailing in a direction E.37^{0}S. at the rate of 8 miles an hour observes a fort in a direction 53^{0} North of East. At 11 a.m. the fort is observed to bear N.20^{0}W., find the distance of the fort from the ship at the first observation.

(6) From the roof of a house 30 feet high the angle of elevation of the top of a monument is 42^{0}7', and the angle of depression of its foot is 17^{0} 59'. Find its height.

Shamash Secondary School

1st.Quarter Exam.

Subject: Arithmetic & Trigonometry Time: 12:00-1:30 p.m.
Class : 4th year Secondary. Date: 8/12/1957

All questions are to be attempted.

(1) State to how many significant digits are the following underlined numbers given ?

My expected profit from my business in the year 1960 is £ 8500. I have to pay my landlord with whom I have just concluded a 10 years agreement £ 1250 per annum. I have to pay my assistant a fixed sum of £ 500 per annum plus a commission of 0.5 per cent on my turnover. His earning from commission may amount to £ 650 per annum. My business premises measures 19.10m, by 25.00 m.

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Shamash Secondary School

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Shamash Secondary School

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Make-up Examination

Fourth Year Secondary

Trigonometry

1/2/[illegible]

Without using tables, find the height and area of an equilateral triangle whose sides are each 7.2 inches long.

2. (i) Find ϕ, if $\sec 17° + \cot 41° = \operatorname{cot}\phi$

(ii) Evaluate in the simplest possible way: $\frac{\operatorname{cosec} 49°}{\operatorname{cosec} 41°}$, $\sin 68° \sec 68°$, $\frac{\operatorname{cosec} 41° \cdot \frac{3}{4}}{\sec 48° 29'}$, $\operatorname{cosec} 75° \cos 15°$

3. a ladder, 26 ft. long, makes an angle of 50° with the ground and leans against a vertical wall. If the top of the ladder slips down 1 ft., how far will the foot of the ladder move?

SHAMASH SCHOOL

FINAL EXAMINATIONS 1953-1954

Subject : Trigonometry — Date : 26/5/54

Class : Fourth year (secondary) — Time : 10:30-12:00

All questions are to be attempted

1. How far down a hill inclined at $7\frac{1}{2}^\circ$ to the horizon must I walk in order to descend a distance of 70 ft. vertically?

2. A is 5 miles due South of a port O. A ship steaming at 10 miles an hour starts from O and steams in a straight line to B 1 mile due West of A. From B the ship steams 37° East of South. Calculate the ship's distance from O at the end of one hour after leaving O.

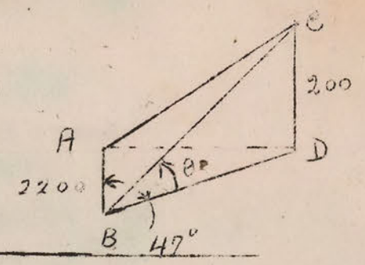
3. Solve the triangle ABC, having given:

$$A = 43^\circ\ 39',\quad C = 17^\circ\ 47',\quad b = 4 \text{ ft.}$$

4. Two points A and B are at sea level, B being due south of A and distant 2200 feet from it. A third point C, which is 200 feet above sea level, is due east of A and its bearing from B is 047° (. (N. 47° E.). Find the horizontal distance between B and C and the angle of elevation of C from B, correct to the nearest 10 feet.

$\theta = ?$

$BD = ?$



Lecturer : Abdullah Obadiah

Class: 4th year Secondary — 1st Quarter Exam.

Subject: Arithmetic + Trigonometry — January 25, 1957

TIME ALLOWED: 90 MINUTES

Attempt six questions only:

(1) a) Define sin A, cos A and tan A where A is an acute angle.

b) Prove sin A = cos (90°-A)

c) Find from first principles sin 30° and tan 45°.

(2) a) Find from tables sin 55° 57' and cos 60° 57'

b) Given tan A = 2.2372 find A from tables.

(3) A B C D is a rectangle in which A B is 15 in. and A D is 8 in. Calculate the angle B A C. The rectangle is held in a vertical plane with A B inclined to the horizontal at 40° and with B C D lower than A.

Calculate the depth of B below A.

(4) a) Divide £117 5s 3d by 49.

b) Express the following as decimals of a pound

12s 3d

18s 9d

9s 8d

(5) The area of a rectangular field is 2 acres and its breadth is 88 yds. Find the perimeter of the field and the length of a diagonal correct to the nearest yard.

(6) What is the correct time when the two hands of a clock coincide between 8 and 9 o'clock.

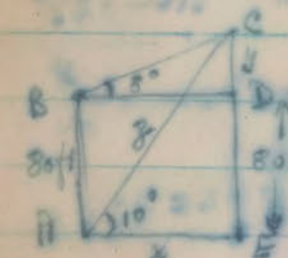
(7) a) A class of 30 students; 12 of whom weigh 9 st. 2 lb each, 8 weigh 8 st. 4 lb. each, 7 weigh 8 st. 1 lb. each and the rest 10 st. 4 lb, 10 st. 6 lb. and 10 st. respectively. Find the average weight of a student to the nearest lb.

b) Define the gallon and state how many pints it contains.

Monthly Exam. April 6/4/5[illegible]

[illegible]ject: Trigonometry Class: Fourth year

1. From the base of a tower, 80 ft. high, the angle of elevation of a distant point is 10°, while when viewed from the top of the tower its elevation is 8°. Find the distance of the point from the base [illegible] tower and its height above the base.



$\frac{x}{y} = \tan 82° \therefore x = y \tan 82°$

$\frac{x}{y+80} = \tan 80° \therefore x = y \tan 80° + 80 \tan 80°$

$\therefore y \tan 82° = y \tan 80° + 80 \tan 80°$

$\therefore y = \frac{80 \tan 80°}{\tan 82° - \tan 80°} = \frac{80 \times 5.6713}{7.1154 - 5.6713}$

$\therefore y = \frac{453.7040}{1.4441} = 314.18$ ft $= 314$ correct to the nearest foot

$\therefore$ [illegible] $= 314 + 80 = 394$ ft Ans. I

$\frac{394}{z} = \sin 10 \quad \therefore z = \frac{394}{\sin 10°} = 394 \csc 10 = 394 \times 5.7588 = 2268.8472$ ft

$= 2270$ ft. correct to the nearest foot.

Ans. II

$\frac{g}{30} = \sin 75° \quad \therefore g = 30 \sin 75°$

$\frac{h}{30} = \sin 30° \quad \therefore h = 20 \sin 30°$

$\therefore g + h = 20 \sin 75° + 20 \sin 30° = 10 (3 \sin 75° + 2 \sin 30°)$

$\therefore g + h = 10 (3 \times 0.9659 + 2 \times \frac{1}{2}) = 10 (2.8977 + 1) = 38.977$ yds

$\frac{x}{30} = \cos 15° \quad \therefore x = 30 \cos 15° = 30 \times 0.2588$

$\frac{y}{20} = \cos 30° \quad \therefore y = 20 \cos 30° = 20 \times 0.8660$

$x + y = 30 \cos 15 + 20 \cos 30 = 10 (3 \times \cos 15 + 2 \cos 30) = 10 (3 \times 0.9659 + 2 \times 0.866)$

$\therefore x + y = 10 (2.7764 + 1.7320) = 10 \times 2.5084 = 25.084$ yds

$\therefore \tan \theta = \frac{x+y}{g+h} = \frac{25.084}{38.977} = 0.6436$

$\therefore \theta = 32° 46'$ Ans. $\quad \therefore$ bearing $= N 22° 46' E$ Ans.

$\angle ACD = 20°$

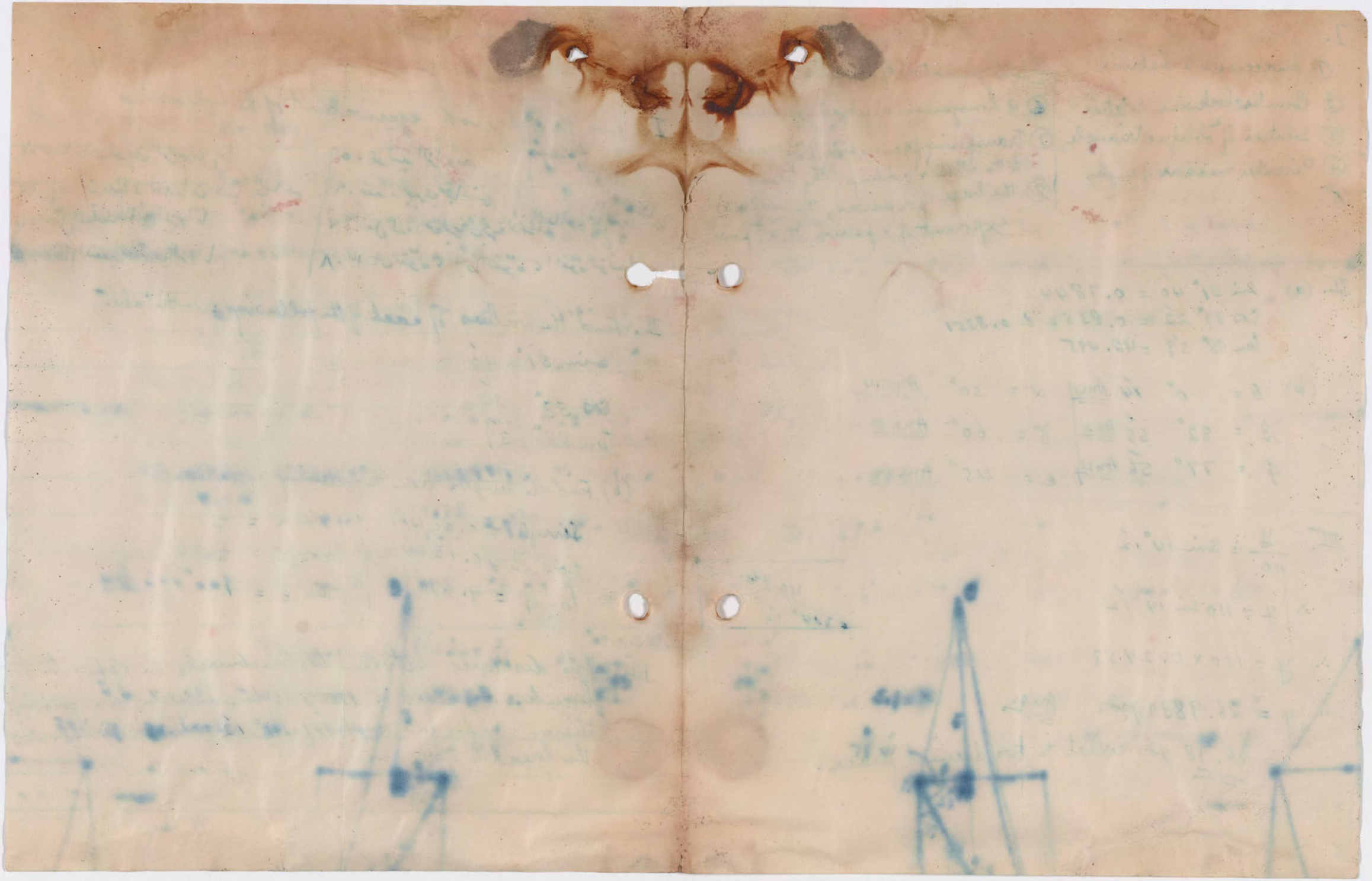
$\frac{x}{14} = \sin 20° \quad \therefore x = 14 \sin 20°$

$\therefore AB = 14 + x = 14 + 14 \sin 20° = 14 (1 + \sin 20°)$

$= 14 (1 + 0.342) = 14 \times 1.342 = 18.788$ in.

$= 18.79$ in. correct to the nearest 0.01 in.

Ans.



I.

① Inaccessible distances
② Counterclockwise rotation
③ Solution of oblique triangle
④ Circular measure of angles
⑤
⑤ The generation of an angle
⑥ a homogeneous algebraic Expression
⑦ Transposing from one side of the equation to the other + adding like terms
⑧ the base of a power, the index or exponent of a power, the k^{th} power

II. (a) $\sin 51^\circ 40' = 0.7844$
$\cos 33^\circ 22' = 0.8352$ & 0.8351
$\tan 88^\circ 39' = 42.495$

(b) $\theta = 0^\circ 14'$ Ans. 1 | $\alpha = 30^\circ$ Ans. 4
$\beta = 82^\circ 55'$ Ans. 2 | $\gamma = 60^\circ$ Ans. 5
$\phi = 77^\circ 56'$ Ans. 3 | $\epsilon = 45^\circ$ Ans. 6

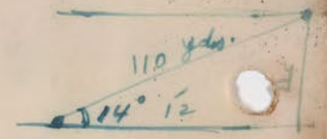
III. $\frac{y}{110} = \sin 14^\circ 12'$

$\therefore y = 110 \sin 14^\circ 12'$

$\therefore y = 110 \times 0.2453$

$\therefore y = 26.9830$ yds Ans.

$= 26.98$ yds correct to two decimal places.



Trigonometry Exam. 13/12/54

Fourth Year

I. Give the English equivalent of the following

١. مسافات لا يمكن الوصول إليها
٢. دوران عكس دوران عقارب الساعة
٣. حل المثلث المائل الزوايا
٤. نظام التقدير الدائري للزوايا
٥. توليد الزاوية
٦. مقدار جبري متجانس
٧. نقل الحدود من طرف إلى الطرف الآخر للمعادلة وجمع الحدود المتشابهة
٨. النسبة بين القوة ، أس القوة ، القوة النونية

II. (a) Find the values of each of the following from the tables

$\sin 51^\circ 40'$

$\cos 33^\circ 22'$

$\tan 88^\circ 39'$

(b) Find the angles from the following equations:

$\sin \theta = 0.00141$ | $\sin \alpha = \frac{1}{2}$
$\cos \beta = 0.1234$ | $\cos \gamma = \frac{1}{2}$
$\tan \phi = 4.6789$ | $\tan \epsilon = 1$

III. The distance between two landmarks on opposite banks of a river is 110 yards, and the line joining them makes an angle of $14^\circ 12'$ with the banks. Find the breadth of the river.

SHAMASH SCHOOL

FINAL EXAMINATIONS 1953-1954

Subject : Trigonometry — Date : 26/5/54

Class : Fourth year (secondary) - — Time : 10:30-12:00

All questions are to be attempted

1. How far down a hill inclined at $7\frac{1}{2}^\circ$ to the horizon must I walk in order to descend a distance of 70 ft. vertically?

2. A is 5 miles due South of a port O. A ship steaming at 10 miles an hour starts from O and steams in a straight line to B 1 mile due West of A. From B the ship steams 37° East of South. Calculate the ship's distance from O at the end of one hour after leaving O.

3. Solve the triangle ABC, having given:

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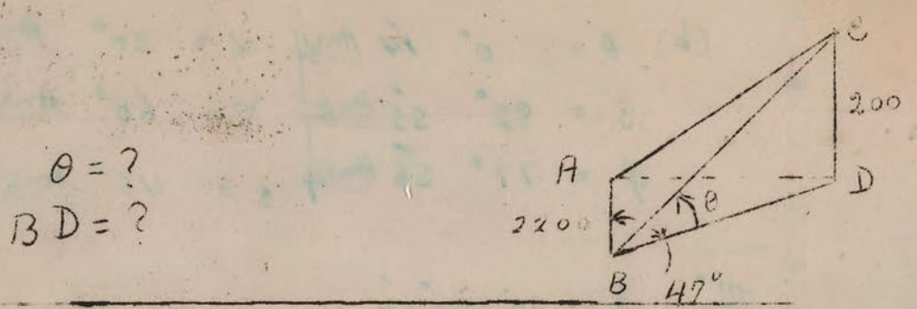
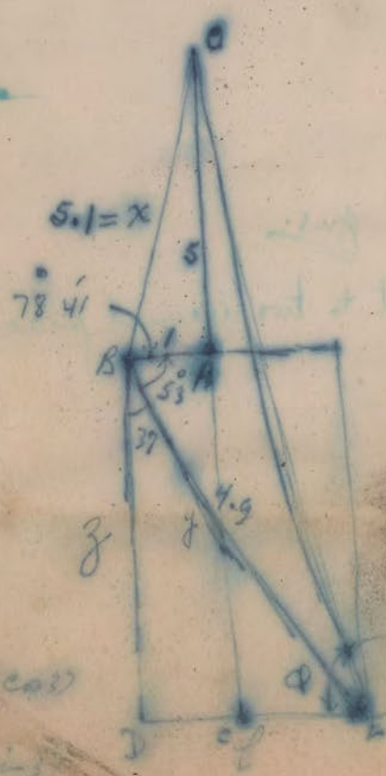
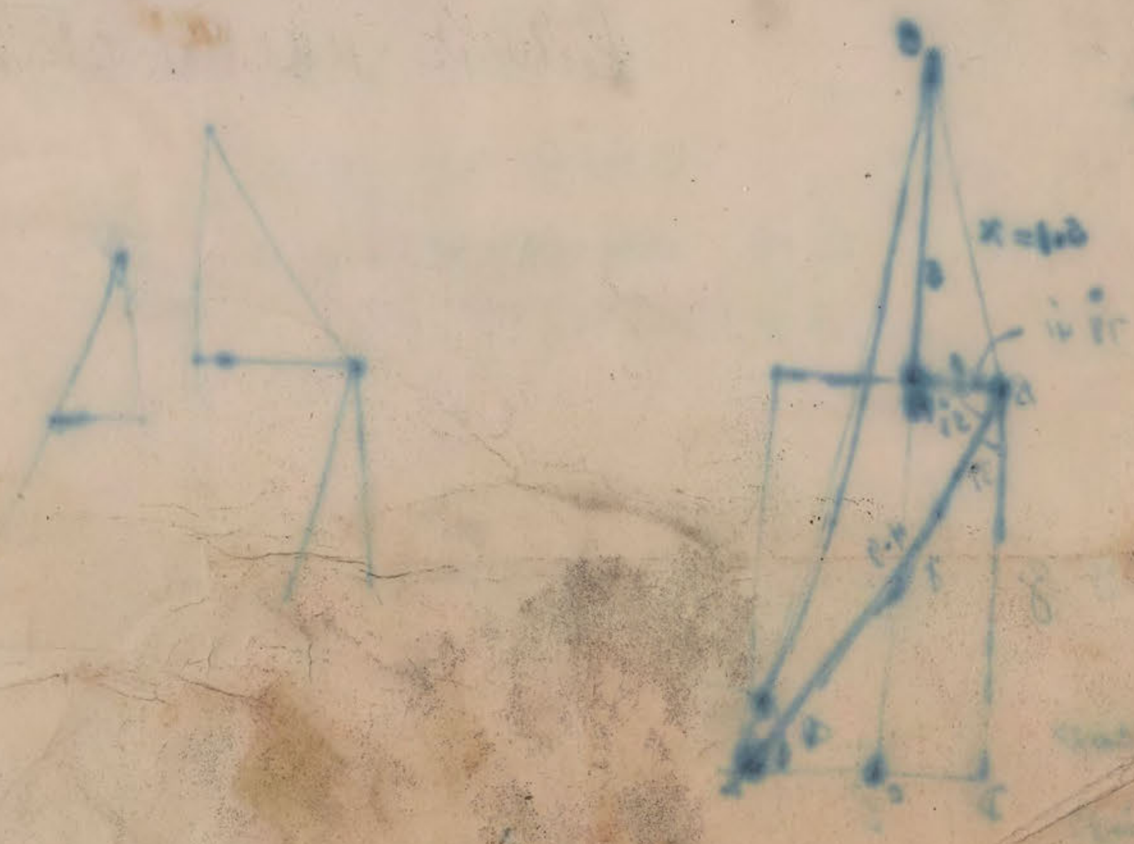


Lecturer : Abdullah Obadiah





[illegible] INY S[illegible]OL
INTE[illegible]EDIATE & PRIMARY
Baghdad
Telephone No. 91693

مدرسة فرنك [illegible]
متوسطة وابتدائية
بغداد
رقم التلفون ٩١٦٩٣

No ....   الرقم ....

Date....................   التاريخ ....

١ـ ترجم الى الانكليزية :

حكى أبو جعفر محمد بن الفضل الصخيري قال : كان في بلدنا عجوز صاحبة كثرة الصيام والصلاة وكان لها ابن صيرفي منهمكاً على الشرب واللعب . وكان يتشاغل بدكانه اكثر نهاره ثم يعود الى منزله فيخبئ كيسه عند والدته ويمضي فيبيت في مواضع يشرب فيها فعني بعض اللصوص على كيسه ليأخذه فجاء وراءه فدخل الى الدار وهو لا يعلم فاختبأ فيها وسلم هو كيسه الى أمه وخرج وبقيت هي وحدها في الدار وكان لها في دارها بيت مؤزر بالساج عليه باب من حديد تجعل قماشها فيه والكيس مخبأت الكيس فيه خلف الباب وجلست فأفطرت بين يديه فقال اللص الساعة تقفله وتنام وانزل واقلع الباب وآخذ الكيس ، فلما أفطرت قامت تصلي ومدت الصلاة ومضى نصف الليل وتحير اللص وخاف أن يدركه الصبح فطاف في الدار فوجد إزاراً جديداً وبخوراً فاتزر بالازار وأوقد البخور ،

٢ـ اكتب معاني هذه الكلمات بالانكليزية :ـ

الطفيلي ـ الزنادقة ـ ولية ـ الورطة ـ المقلى ـ أمة (أُرَيمة) ـ محدث ـ كرهاً ـ
الأمان ـ الإسناد ـ

3. Translate into Arabic:

Summer Holidays

Then comes July, and with it examinations, but these are soon finished, and with them ends the school year. Boys and [illegible] have nearly two months holiday before them as [illegible] school by train and car to return home [illegible] s and mothers [illegible]

The summer holidays [illegible]

for most children. The weather is usually good, so that one can spend most of one's time playing in the garden, if one lives in the country, out in the woods and fields. Even if one lives in a big town, one can usually go to a park to play.

The best place for a summer holiday, however, is the seaside. Some children are lucky enough to live near the sea, but for the others who ~~so~~ do not, a week or two at one of the big seaside towns is something which they will talk about for the whole of the following year.

Shamash Secondary School

1st. Quarter Exam.

Subject: Arithmetic & Trigonometry — Time: 12:00-1:30 p.m.
Class: 4th year Secondary. — Date: 8/12/1957

All questions are to be attempted.

(1) State how many significant digits are the following and in numbers given ?

My expected profit from my business in the year 1960 is £ 8500. I have to pay my landlord with whom I have just concluded a 10 years agreement £ 1250 per annum. I have to pay my assistant a fixed sum of £ 500 per annum plus a commission of 0.5 per cent on my turnover. His earning from commission may amount to £ 650 per annum. My business premises measures 19.10m, by 25.00 m.

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(6) From the roof of a house 30 feet high the angle of elevation of the top of a monument is 42°7', and the angle of depression of its foot is 17° 59'. Find its height.

Shamash School
Arithmetic Quiz

Subject: Trigonometry
Class: 4th

Date: 25/1/55
Time: ~~40 minutes~~ 1 hour.

All questions are to be attempted.

1. (a) Find θ, ~~α~~ ϕ and ψ when:
$\cot\theta = 2.4363$, $\sec\phi = 3.1241$, $\operatorname{cosec}\psi = 4.0213$
$22^\circ\ 19'$ $71^\circ\ 20'$ $14^\circ\ 24'$

(b) Solve the triangle ABC, given that: $C = 90^\circ$, $B = 71^\circ\ 31'$, $b = 76$ in.
$A = 18^\circ\ 29'$ $a = 25.4068$ $c = 81.19$

2. (a) Evaluate as shortly as possible $\frac{1}{\sin 19^\circ}$, $\frac{1}{\cos 29^\circ}$, $\frac{1}{\tan 42^\circ}$,
3.0713 1.1430 1.1106

$\frac{\cos 17^\circ}{\sin 17^\circ}$, $\frac{\sin 42^\circ}{\sin 48^\circ}$, $\frac{\cos 63}{\cos 27}$
3.2709 0.9004 0.5095

(b) If $\tan 9^\circ + \tan 31^\circ = \tan\theta$, Find θ using tables
$0.3443 + 0.6009 = 0.9452 \therefore \theta = 43^\circ\ 23'$

3. The sun is due ~~west~~ W. at an elevation of 25°.
(a) Find the length of the shadow thrown on the ground by a vertical pole 30 ft high. $30 \times 2.1445 = \frac{30}{.4663} = 64.335$ ft
(b) Find the height of this shadow on a vertical wall running N. + S. and 10 yds away from the pole. 16.08 ft. $= 34.335 \times 0.4663$